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Aggregation over commodities: An application of the Generalized Composite Commodity Theory in a US food demand system analysis

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by

Yanrong Wang

A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Economics

Major Professor: J. Arne Hallam

Iowa State University

Ames, Iowa

1997

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This is to certify that the Master's thesis of

Yanrong Wang

has met the thesis requirements of Iowa State University

Signatures have been redacted for privacy

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1 INTRODUCTION

Fundamentals of the consumer demand system

The neoclassical theory of consumer demand is well documented in a number of sources. Neoclassical theory pertains to a single individual's consumption decision at a given point of time. The fundamentals of the theory include the "utility function, the commodity set and the axiom concerning the order of preference" (Raunikar, 1987, p. 4).

The utility function

The utility function $U = U(q_1, q_2, \cdots, q_n)$ measures the level of satisfaction consumer experiences from the consumption of the commodity bundle $(q_1, q_2, \cdots, q_n)$. Generally, the utility function is denoted as

$$U = u(\underline{q})$$

where $\underline{q} = (q_1, q_2, \cdots, q_n)'$ is an $n \times 1$ vector.

The utility function is assumed to be strictly increasing, strictly quasi-concave and twice differentiable.

The commodity set

The commodity set has three properties,

1. non-negativity property.

2. divisibility property.

3. unboundedness property.

Preference axioms

Preference axioms include comparability, antisymmetry, transitivity, reflexivity, local non-satiation, continuity, convexity, monotonicity and differentiability. A detailed discussion can be found in Deaton (1980).

Derivation of the demand system

Marshallian demand function

The Marshallian demand system is derived from utility maximization subject to the budget constraint. The corresponding problem is

$$maxU = U(\underline{q})$$

s.t.

$$\begin{aligned} m &\geq \underline{p}'\underline{q} = \sum_{i=1}^{n} p_i q_i \\ q_i &\geq 0 \text{ for } i = 1, 2, \cdots, n \end{aligned}$$

where $\underline{p} = (p_1, p_2, \cdots, p_n)'$ and m is the income or total budget.

Thus, the Lagrangian is

$$L = U(\underline{q}) + \lambda(m - \underline{p}'\underline{q})$$

The corresponding Kuhn-Tucker conditions are

$$\begin{aligned} \frac{\partial L}{\partial q_i} &= \frac{\partial U}{\partial q_i} - \lambda p_i \leq 0 \ (= 0 \text{ if } q_i > 0) \text{ for } \forall i = 1, 2, \cdots, n \\ \frac{\partial L}{\partial \lambda} &= m - \underline{p}'\underline{q} \geq 0 \ (= 0 \text{ if } \lambda > 0). \end{aligned}$$

Under strong monotonicity,

$$\frac{\partial U}{\partial q_i} > 0 \text{ for } \forall i = 1, 2, \cdots, n.$$

So $\lambda > 0$, which implies $m - \underline{p}'\underline{q} = 0$ for the optimal bundle. Hence the optimal set is on the boundary of budget line.

Given $\underline{p}$, m and utility function, we can solve for indirect function (Marshallian demand function)

$$q_i^* = q_i(p_1, p_2, \cdots, p_n, m) = q_i(\underline{p}, m) \text{ for } \forall i = 1, 2, \cdots, n.$$

The indirect utility function is

$$U(\underline{q}^*) = U(q_1(\underline{p}, m), q_2(\underline{p}, m), \cdots, q_n(\underline{p}, m)) = V(\underline{p}, m).$$

One important identity related to indirect utility function is Roy's identity which states

$$q_i^* = -\frac{\partial V(\underline{p}, m)/\partial p_i}{\partial V(\underline{p}, m)/\partial m} \text{ for } \forall i = 1, 2, \cdots, n.$$

Hicksian demand function

The Hicksian demand function, or compensated demand function, is developed based on the duality concept. That is, we want to minimize the cost at a given utility level, say $\overline{u}$. So, the problem is

$$min\ \underline{p}'\underline{q} = \sum_{i=1}^{n} p_i q_i$$

s.t.

$$U(\underline{q}) = \overline{u}.$$

Thus we can get a Hicksian demand function

$$\overline{q}_i = h_i(\underline{p}, \overline{u}) \text{ for } \forall i = 1, 2, \cdots, n$$

and the cost function

$$C(\underline{p}, \overline{u}) = \sum_{i}^{n} h_i(\underline{p}, \overline{u}) \cdot p_i.$$

By Shephard's Lemma

$$\overline{q}_i = \frac{\partial C(\underline{p}, \overline{u})}{\partial p_i} \text{ for } \forall i = 1, 2, \cdots, n.$$

Restrictions on the demand system

The concept of duality tells us that

$$\begin{aligned} V(\underline{p}, C(\underline{p}, \overline{u})) &\equiv \overline{u} \\ C(\underline{p}, V(\underline{p}, m)) &\equiv m. \end{aligned}$$

As a result, we can show the Slutsky equation

$$\frac{\partial q_i(\underline{p}, m)}{\partial p_j} = \frac{\partial h_i(\underline{p}, u^*)}{\partial p_j} - q_j^* \frac{\partial q_i(\underline{p}, m)}{\partial m}.$$

If we define

$$\begin{aligned} \varepsilon_i &= \frac{\partial q_i}{\partial m} \frac{m}{q_i} \text{ for } \forall i = 1, 2, \ldots, n \\ \varepsilon_{ij} &= \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} \text{ for } \forall i, j = 1, 2, \ldots, n \\ \eta_{ij} &= \frac{\partial h_i}{\partial p_j} \frac{p_j}{h_i} \text{ for } \forall i, j = 1, 2, \ldots, n \\ w_i &= p_i q_i m \text{ for } \forall i = 1, 2, \cdots, n, \end{aligned}$$

then the elasticity version of the Slutsky equation is

$$\varepsilon_{ij} = \eta_{ij} - w_j \varepsilon_i.$$

The axioms of monotonicity, convexity and differentiability of the utility function ensure a unique solution for q_i as a function of $\underline{p}$ and m. Moreover, the Hession matrix

$$H = \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ U_{21} & U_{22} & \cdots & U_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ U_{n1} & U_{n2} & \cdots & U_{nn} \end{bmatrix}$$

is a negative definite matrix where

$$U_{ij} = \frac{\partial^2 U}{\partial q_i \partial q_j} \text{ for } \forall i, j = 1, 2, \cdots, n.$$

It is obvious that the demand system derived above should also satisfy a number of restrictions, namely, the homogeneity restriction, the adding-up restriction and the symmetry restriction.

1. Adding-up restriction.

$$\sum_{i=1}^{n} p_i q_i = m$$

or equivalently

$$\sum_{i}^{n} w_i = 1.$$

If expressed in terms of elasticities, adding-up implies

(a) Engel aggregation condition

$$\sum_{i=1}^{n} w_i \varepsilon_i = 1.$$

(b) Cournot aggregation condition

$$\sum_{i=1}^{n} w_i \varepsilon_{ij} = -w_j \text{ for } \forall j = 1, 2, \cdots, n.$$

2. Homogeneity condition.

$$q_i(a\underline{p}, am) = q_i(\underline{p}, m)$$

where a is a constant.

Similarly, we can write this restriction in terms of elasticities

$$\sum_{j=1}^{n} \varepsilon_{ij} + \varepsilon_i = 0 \quad \text{for} \quad \forall i = 1, 2, \cdots, n.$$

3. Slutsky symmetry conditions.

$$\varepsilon_{ij} = \varepsilon_{ji} \frac{w_j}{w_i} + w_j(\varepsilon_j - \varepsilon_i) \text{ for } \forall i, j = 1, 2, \cdots, n.$$

In this paper, a food demand system is first estimated with only the adding-up restrictions.[1] Then, the restricted models are estimated after the homogeneity and symmetry restrictions are imposed.

Aggregation issues

The neoclassical theory of consumer behavior is based on a single individual, thus is micro in nature. In real life, with millions of commodities and consumers, aggregation is unavoidable in empirical studies. In essence, aggregation theory transforms micro-relationships to macro-relationships (Thiel, 1954).

Typically, there are three types of aggregation: aggregation over individuals, aggregation over time and aggregation over commodities. Aggregation over individuals and aggregation over commodities are the two most commonly discussed in the literature. The former comes naturally because usually, economists are more interested in the behavior of the whole market than in that of an individual consumer. Therefore, demand analysis generally consists of the aggregate demand for a number of consumers. The purpose of aggregation over commodities is to treat aggregated data as if they were related to individual commodities.

Both of aggregation topics need to be treated at a basis of a satisfactory theoretical and empirical level. Deaton (1980) pointed out that aggregation is one of the topics that are basic to an understanding of consumer behavior.

[1] Since the shares in the data set add up to 1, the adding-up condition holds implicitly.

In this paper, we address the issue of aggregation over commodities. Chapter 2 and Chapter 3 develop the theoretical framework. Chapter 4 discusses the Almost Ideal Demand System (AIDS) model that is used in the model specification for the food demand system. Chapter 5 deals with the data used in the estimation. Interpretation and analysis of the empirical study are given in Chapter 6.

2 AGGREGATION OVER COMMODITIES

In the utility function we derived in Chapter 1, we defined n commodities in the function. In reality, there are literally millions of different goods in the market. So, we can only hope to deal with a relatively small number of the commodities. Usually, in the literature, a demand system based on groups of commodities is widely applied. One key question remains: is it justified to use commodity groups instead of individual goods? In other words, can we treat a group of goods as if they were a single commodity?

In the rest of this chapter, we will investigate the theorems pertaining to this topic.

Separability

Separability is one of the widely used concepts in the literature of demand system estimation. The idea is in the context of separability of preferences. That is the commodities can be partitioned into groups, for instance the food group, the clothing group, the transportation group, the durable group, ... , so that preference within groups can be described independently of the quantities in other groups.

Notations and implications of separability

There are different notations on separability. Four commonly used are as follows.

1. Direct Weak Separability (DWS).

Direct Weak Separability (DWS) holds if utility can be written in terms of sub-utility functions, which are dependent on a subset of goods. That is

$$U = U^0(U_1(\underline{q}^1), U_2(\underline{q}^2), \ldots, U_s(\underline{q}^s))$$

where $\underline{q}^1, \underline{q}^2, \ldots, \underline{q}^s$ represent a partition of the $n \times 1$ vector of goods, s is the total number of separable groups, $s < n$. Sub-utility functions $U_g(\underline{q}^g), g = 1, 2, \ldots, s$, have the same properties as a normal utility function. DWS has three main implications on the system.

(a) The marginal rate of substitution (MRS) between two goods in a separable group is independent of consumption of goods outside the group. That is , for goods i, j, if $(i, j) \in I_g$ then

$$\begin{aligned} MRS_{ij} &= \frac{\partial U / \partial q_i}{\partial U / \partial q_j} \\ &= \frac{\frac{\partial U^0}{\partial U_g} \frac{\partial U_g}{\partial q_i}}{\frac{\partial U^0}{\partial U_g} \frac{\partial U_g}{\partial q_j}} \\ &= \frac{\partial U_g / \partial q_i}{\partial U_g / \partial q_j}. \end{aligned}$$

Hence

$$\frac{\partial MRS_{ij}}{\partial q_k} = 0 \text{ for } \forall k \notin I_g.$$

(b) There exist conditional functions with the demand of a good depending only on the prices of the separable group and on the income allocated to the group

$$q_i = q_i^c(\underline{p}^g, x_g) \text{ for } \forall i \in I_g, g = 1, 2, \ldots, s$$

It is obvious now that the separability is closely related to the two-stage budget. In the first stage, income m is allocated to the s groups, say, x_1,x_2,$\cdots$,x_s, based on price vector $\underline{p}_{n \times 1}$; in the second stage, x_g is allocated within group based on $\underline{p}^g, x_g,\ g = 1, 2, \cdots, s$. Prices outside group g will affect demand in this group only through x_g.

(c) DWS imposes additional restrictions on elasticities. Under DWS, the Slutsky terms for goods belonging to different groups are proportional to the income elasticity.

$$\eta_{ik} = w_k \theta_{gs} \varepsilon_i \varepsilon_j$$

for $\forall i \in I_g, k \in I_s, g \neq s$ where

$$\begin{aligned} \eta_{ik} &= \frac{\partial h_i}{\partial p_k} \frac{p_k}{h_i} \\ \varepsilon_i &= \frac{\partial q_i}{\partial x} \frac{x}{q_i} \\ \varepsilon_k &= \frac{\partial q_k}{\partial x} \frac{x}{q_k} \\ \theta_{gs} &= \text{a constant.} \end{aligned}$$

A detailed proof can be found in Deaton (1980).

2. Directly strong separability (DSS)

Directly strong separability (DSS) is also called block-wise additivity. It imposes more structure on the preference. DSS can be written as

$$U = U_1(\underline{q}^1) + U_2(\underline{q}^2) + \ldots + U_s(\underline{q}^s).$$

It is obvious now that DSS implies DWS. However, DSS imposes stronger restrictions than DWS does. DSS requires

$$\frac{\partial MRS_{ij}}{\partial q_k} = 0 \text{ for } \ \forall i \in I_g, \forall j \in I_s, \forall k \notin I_s \cup I_g.$$

3. Implicit separability (quasi separability).

Implicit separability is expressed in terms of the cost function.

$$C = C^0(c_1(\underline{p}^1, u), c_2(\underline{p}^2, u), \ldots, c_s(\underline{p}^s, u))$$

4. Indirectly weak separability.

   Indirectly weak separability describes the demand system in terms of the indirect utility function

$$V = V^0(V_1(\underline{p}^1, m), V_2(\underline{p}^2, m), \ldots, V_s(\underline{p}^s, m)).$$

Aggregation over commodities under separability

To aggregate over commodities in the context of separability, we need further assumption to ensure the consistency with constrained utility maximization. The studies by Gorman (1959), and Bieri and de Janvry on two-stage budgeting serve as a theoretical framework.

Gorman's theory of two-stage budgeting suggests that given a DWS utility function partitioning of commodities into s groups, price aggregation is possible if and only if

1. the utility function is strongly separable (DSS) with a Generalized Gorman Polar Form (GGPF);

2. or the sub-utility functions are homothetic.

In the first case, we have an additive utility function,

$$U = U_1(\underline{q}^1) + U_2(\underline{q}^2) + \ldots + U_s(\underline{q}^s).$$

The aggregation functions $U_g(\underline{q}^g)$ have the following GGPF form,

$$U_g(\underline{q}^g) = F_g(\frac{x_g}{B_g(\underline{p}^g)}) + A_g(\underline{p}^g) \text{ for } \; g = 1, 2, \cdots, s$$

where A_g, B_g are functions of $\underline{p}^g$.

Thus the utility maximization problem is

$$max_{x_1, x_2, \ldots, x_s} \sum_{g=1}^{s} F_g(\frac{x_g}{B_g(\underline{p}^g)}) + \sum_{g=1}^{s} A_g(\underline{p}^g)$$

s.t.

$$x_1 + x_2 + \ldots + x_s = m.$$

Since $A_g(\underline{p}^g)$'s don't appear in the first order conditions (FOC), we can solve for

$$x_g^* = x_g(B_1, B_2, \ldots, B_s, x).$$

In other words, we can find group expenditures as functions of the group price indices B_i's and the aggregated income x.

In the second case, we have DWS utility function,

$$U = U_0(U_1(\underline{q}^1), U_2(\underline{q}^2), \ldots, U_s(\underline{q}^s)).$$

The sub-utility functions are homothetic and have the form

$$U_g = V_g(\underline{p}^g, x_g) = \frac{x_g}{b_g(\underline{p}^g)}.$$

Thus the corresponding utility maximization problem is

$$max_{x_1, x_2, \ldots, x_s} U^0(\frac{x_1}{b_1(\underline{p}^1)}, \frac{x_2}{b_2(\underline{p}^2)}, \ldots, \frac{x_s}{b_s(\underline{p}^s)})$$

s.t.

$$x_1 + x_2 + \cdots + x_s = m.$$

Therefore,

$$x_g^* = x_g(b_1, b_2, \ldots, b_s, x).$$

Therefore, in both cases we can treat a group of goods as if they were a single commodity.

The concept of separability is appealing because it suggests the idea of a "utility tree". In empirical work, separability can be easily tested by estimating models for individual goods without separability, then test the elasticity restrictions implied by separability. However, without separability, each of the systems needs to include hundreds of goods, which leads to less degrees of freedom in the system. Moreover, even when there are sufficient degrees of freedom, data problems such as multicollinearity among the prices cause the resulting test to have little power. Barnett and Choi (1989)'s Monte-Carlo study showed that standard tests tended to fail to reject separability even when the data constructed from utility functions that were far from separable. This "difficult to reject" property of separability is probably one reason that it is widely applied in empirical studies.

However, we can see that the two cases suggested by Gorman's theory both impose quite implausible restrictions on the demand system. In first case, elasticities are restricted by the additive structure of the utility function, while in the second case, the sub-utility functions are homothetic which implies the expenditure term is linear in the conditional demand system. Therefore, separability is not fundamentally credible as representation of the behavior and phenomena we want to understand and explain.

Hicks-Leontief composite commodity theory

Hicks-Leontief's composite commodity theory (Hicks and Leontief, 1936) has been the only alternative to separability for aggregation over commodities before Lewbel (1996) proposed the Generalized Composite Commodity Theory (GCCT).

Hicks-Leontief's theorem states that if the prices of a group of goods change in the same proportion, then the group of goods behaves just as if they were a single commodity.

If we denote $\underline{q} = (\underline{q}^1, \underline{q}^2)$, where $\underline{q}^1$ is the group of goods whose prices move proportionally. Let $\underline{p}^1, \underline{p}^2$ be the corresponding price vectors, subscript 0 and 1 denote for base year and current year respectively. Then, by assumption,

$$\underline{p}_1^1 = a\underline{p}_0^1$$

where a is the proportion of price change for the group of goods whose prices move absolutely together. Therefore, the utility maximization problem is

$$maxU(\underline{q}^1, \underline{q}^2)$$

s.t.

$$\underline{p}_1^{1'}\underline{q}_1^1 + \underline{p}_1^{2'}\underline{q}_1^2 = m.$$

Or equivalently

$$a\underline{p}_0^{1'}\underline{q}_1^1 + \underline{p}_1^{2'}\underline{q}_1^2 = m$$

or

$$aQ + \underline{p}_1^{2'}\underline{q}_1^2 = m$$

where $Q = \underline{p}_0' q_1^1$ is the aggregated quantity.

The resulting indirect utility function is

$$V = V(\underline{p}_1^1, \underline{p}_1^2, m) = V_1(a, \underline{p}_1^2, m).$$

To satisfy the aggregation property, we need to show that we can treat V_1 as an indirect function. It is sufficed to show that if V_1 satisfies Roy's identity, i.e., we need to check if

$$Q^* = -\frac{\partial V_1/\partial a}{\partial V_1/\partial m}.$$

Given

$$\frac{\partial V_1}{\partial a} = \sum_i \frac{\partial V}{\partial p_1^{1i}} p_0^{1i},$$

we know

$$q_1^{1i*} = -\frac{\partial V/\partial p_1^{1i}}{\partial V/\partial m} \text{ by Roy's identity}$$

and

$$\frac{\partial V_1}{\partial m} = \frac{\partial V}{\partial m},$$

thus,

$$-\frac{\partial V_1/\partial a}{\partial V_1/\partial m} = \sum_i q_1^{1i*} p_0^{1i} = Q^*.$$

Hence, if the prices of a group of goods move absolutely synchronously, we can treat the group as a commodity. However, even though prices of related goods tend to move in the same direction, Hicks-Leontief's theorem requires them to move exactly proportionally, which is clearly not supported by empirical evidence.

3 THE GENERALIZED COMPOSITE COMMODITY THEOREM

In a recent paper, Arthur Lewbel (1996) proposed the Generalized Composite Commodity Theorem (GCCT) by relaxing Hicks-Leontief's assumptions.

If we denote $\rho_i = \log p_i - \log P_I$, for $i \in I$, i denotes an individual good, $i = 1, 2, \cdots, n$, I is the group, $I = 1, 2, \cdots, s$,p_i is the price index for ith good, P_I is a price index for the Ith group. Hicks-Leontief's theorem requires the ρ_i's to be constants for each of the groups. Lewbel relaxed this assumption by allowing ρ_i's to vary over time and be independently distributed of group prices $\underline{P}$ and m. In other words, it only requires that the relative price of a good within a group be uncorrelated with the group prices. This assumption is more realistic than Hicks-Leontief's. In the food demand system estimated in this paper, we will check the validity of this assumption by investigating the prices of individual commodities and the groups.

In the rest of this chapter, we will review the theorem in detail.

Notation

The notation we are going to use in this paper is as follows.

1. $z = \log m$ is the log of the consumer's total consumption expenditure.

2. $r_i = \log p_i$ is the log of the price of commodity i, $i = 1, 2, \cdots, n$, n is total number of goods. $\underline{r}$ is the vector of r_i or $(r_1, r_2, \cdots, r_n)'$.

3. $R_I = \log P_I$ is the log of the price of group I, $I = 1, 2, \cdots, s$, s is the total number of groups. $\underline{R}$ the vector of R_I, or $(R_1, R_2, \cdots, R_s)'$.

4. $\rho_i = \log \frac{p_i}{P_I} = r_i - R_I$ is the difference between the individual price and the group price, where $i \in I$, or the relative price of goods i within group I. $\underline{\rho}$ is the vector of ρ_i, or $(\rho_1, \rho_2, \cdots, \rho_n)$'.

5. $\underline{R}^* = \underline{r} - \underline{\rho}$ is the vector of R_i^*, where $R_i^* = R_I$, for $i \in I$. This is because $R_I = r_i - \rho_i$ for $i \in I$.

6. A demand function is integrable if it satisfies adding-up, homogeneity and symmetry conditions; it is rational if in addition, it satisfies negative semi-definiteness.

7. Define share of good i as $w_i = \dfrac{p_i q_i}{m}$ for $i = 1, 2, \cdots, n$. Define share of group I as $W_I = \sum_{i \in I} w_i$, $I = 1, 2, \cdots, s$.

8. $w_i = g_i(\underline{r}, z) + e_i$ where $E(e_i|\underline{r}, z) = 0$, g_i is a function of $\underline{r}$ and z, $i = 1, 2, \cdots, n$.

9. Define $s_{ij} = \dfrac{\partial g_i(\underline{r}, z)}{\partial r_j} + \dfrac{\partial g_i(\underline{r}, z)}{\partial z} g_j(\underline{r}, z)$.

10. Define $G_I^* = \sum_{i \in I} g_i(\underline{r}, z)$ as group I's share expressed in terms of $\underline{r}$ and z.

11. $W_I = G_I(\underline{R}, z) + e_I$ where $E(e_I|\underline{R}, z) = 0$, G_I is a function of $\underline{R}$ and z, $I = 1, 2, \cdots, s$. It can be shown that

$$G_I(\underline{R}, z) = E[G_I^*(\underline{R}^* + \underline{\rho}, z)|\underline{R}, z] = \int G_I^*(\underline{R}^* + \underline{\rho}, z|\underline{R}, z) dF(\underline{\rho})$$

This holds because

$$W_I = G_I(\underline{R}, z) + e_I = \sum_i [g_i(\underline{r}, z) + e_i].$$

Given

$$E(e_i|\underline{R}, z) = 0,$$

$$
\begin{aligned}
G_I(\underline{R}, z) &= E[\sum_i (g_i(\underline{r}, z) + e_i|\underline{R}, z)] \\
&= E(G_I^*(\underline{R}^* + \underline{\rho}, z)|\underline{R}, z).
\end{aligned}
$$

The last equality holds because

$$E(e_i|\underline{R}, z) = 0.$$

12. Define

$$
\begin{aligned}
H_{IJ} &= COV\left[\frac{\partial G_I^*(\underline{R}^* + \underline{\rho}, z)}{\partial z}, G_J^*(\underline{R}^* + \underline{\rho}, z)|\underline{R}, z)\right] \\
&= \int \frac{\partial G_I^*(\underline{R}^* + \rho, z)}{\partial z} \times G_J^*(\underline{R}^* + \rho, z)|\underline{R}, z)dF(\underline{\rho}) \\
&\quad - \frac{\partial G_I(\underline{R} + \underline{\rho}, z)}{\partial z} \times G_J(\underline{R}, z).
\end{aligned}
$$

Let H be the corresponding matrix with H_{IJ} at the (I, J) position.

13. Define

$$
\begin{aligned}
\overline{H_{IJ}} &= COV[G_I^*(\underline{R}^* + \underline{\rho}, z), G_J^*(\underline{R}^* + \underline{\rho}, z)|\underline{R}, z)] \\
&= \int G_I^*(\underline{R}^* + \underline{\rho}, z) G_J^*(\underline{R}^* + \underline{\rho}, z) dF(\underline{\rho}) \\
&\quad - G_I(\underline{R} + \underline{\rho}), z) \times G_J(\underline{R}, z).
\end{aligned}
$$

Let $\overline{H}$ be the corresponding matrix with $\overline{H_{IJ}}$ at the (I, J) position.

14. Define

$$S_{IJ}(\underline{R}, z) = \frac{\partial G_I(\underline{R}, z)}{\partial R_J} + \frac{\partial G_I(\underline{R}, z)}{\partial z} \times G_J(\underline{R}, z).$$

The Generalized Composite Commodity Theorem (GCCT)

The Generalized Composite Commodity Theorem (GCCT) is as follows:

1. Assume the $g_i(\underline{r}, z)$'s are rational demand functions. I.e., they satisfy adding-up, homogeneity, symmetry and negative semi-definiteness.

2. Assume the distribution of the random vector $\underline{\rho}$ is independent of $\underline{R}$ and z,

then, the group demand functions $G_I(\underline{R}, z)$ satisfies the homogeneity and adding-up conditions. Given that H is symmetric, $G_I(\underline{R}, z)$ is symmetric. In addition, if $\overline{H} - H$ is negative semi-definite, then $G_I(\underline{R}, z)$ also satisfies the negative semi-definite property.

Proof

1. Adding-up.

$$\begin{aligned}\sum_I G_I(\underline{R}, z) &= E(\sum_I G_I^*(\underline{R}^* + \underline{\rho}, z)|\underline{R}, z) \\ &= E[\sum_I \sum_{i\in I} g_i(\underline{r}, z)|\underline{R}, z] = E(1|\underline{R}, z) = 1.\end{aligned}$$

2. Homogeneity.

$$\begin{aligned}G_I(\underline{R} - k\underline{1}, z - k) &= E[G_I^*(\underline{R}^* + \underline{\rho} - k\underline{1}, z - k)|\underline{R}, z] \\ &= \int G_I^*(\underline{R}^* + \underline{\rho} - k\underline{1}, z - k)dF(\underline{\rho}) \\ &= \int \sum_{i\in I} g_i(\underline{R}^* + \underline{\rho} - k\underline{1}, z - k)dF(\underline{\rho}) \\ &= \int \sum_{i\in I} g_i(\underline{R}^* + \underline{\rho}, z)dF(\underline{\rho}) \text{ by the homogeneity property of } g_i's \\ &= \int G_I^*(\underline{R}^* + \underline{\rho}, z)dF(\underline{\rho}). \\ &= G_I(\underline{R}, z)\end{aligned}$$

3. Slutsky symmetry.

$$\begin{aligned}E[\sum_{i\in I}\sum_{j\in J} s_{ij}(\underline{r}, z)|\underline{R}, z] &= E\{\sum_{i\in I}\sum_{j\in J}[\frac{\partial g_i(\underline{r}, z)}{\partial r_j} + \frac{\partial g_i(\underline{r}, z)}{\partial z} g_j(\underline{r}, z)]|\underline{R}, z\} \\ &= E[\sum_{i\in I}\sum_{j\in J}\frac{\partial g_i(\underline{r}, z)}{\partial r_j}|\underline{R}, z] \\ &\quad +E[\sum_{i\in I}\sum_{j\in J}\frac{\partial g_i(\underline{r}, z)}{\partial z} g_j(\underline{r}, z)|\underline{R}, z] \\ &= E[\sum_{i\in I}\sum_{j\in J}\frac{\partial g_i(\underline{R}^* + \underline{\rho}, z)}{\partial R_J^* + \rho_j}|\underline{R}, z]\end{aligned}$$

$$
\begin{aligned}
&+E[\sum_{i\in I}\frac{\partial g_i(\underline{R}^*+\underline{\rho},z)}{\partial z}G_J^*(\underline{R}^*+\underline{\rho},z)|\underline{R},z] \\
&= E[\sum_{i\in I}\frac{\partial g_i(\underline{R}^*+\underline{\rho},z)}{\partial R_J}|\underline{R},z] \\
&\quad+\int\frac{\partial G_I^*(\underline{R}^*+\underline{\rho},z)}{\partial z}G_J^*(\underline{R}^*+\rho,z)dF(\underline{\rho}) \\
&= \int\sum_{i\in I}\frac{\partial g_I^*(\underline{R}^*+\underline{\rho},z)}{\partial R_J}dF(\underline{\rho}) \\
&\quad+\int\frac{\partial G_I^*(\underline{R}^*+\rho,z)}{\partial z}G_J^*(\underline{R}^*+\underline{\rho},z)dF(\underline{\rho}) \\
&= \int\frac{\partial G_I^*(\underline{R}^*+\underline{\rho},z)}{\partial R_J}dF(\underline{\rho}) \\
&\quad+\int\frac{\partial G_I^*(\underline{R}^*+\rho,z)}{\partial z}G_J^*(\underline{R}^*+\underline{\rho},z)dF(\underline{\rho}) \\
&= \frac{\partial G_I(\underline{R},z)}{\partial R_J}+\frac{\partial G_I(\underline{R},z)}{\partial z}G_J(\underline{R},z)+H_{IJ}(\underline{R},z) \\
&= S_{IJ}+H_{IJ}.
\end{aligned}
$$

Thus, the asymmetry of S_{IJ} can only come from H_{IJ}. If H_{IJ} is symmetric, then, G_{IJ} is also symmetric.

We will skip the proof for semi-negativity since we are going to focus on the integrability of the demand system rather than rationality in this paper. A detailed proof can be found in Lewbel (1996).

Elasticities under the Generalized Composite Commodity Theorem

In this section, we want to investigate the relationship of group elasticities from the aggregated model and commodity elasticities. This issue arises naturally as elasticity and welfare estimates are of great important to policy making.

As the first step, we want to show how to calculate the elasticities from share equations. Given

$$w_i = \frac{p_i q_i}{m},$$

$$\begin{aligned}
\frac{\partial w_i}{\partial \log p_i} &= \frac{p_i q_i}{m} - \frac{p_i \frac{\partial q_i}{\partial \log p_i}}{m} \\
&= \frac{p_i q_i}{m}(1 - \varepsilon_{ii}) \\
&= w_i(1 - \varepsilon_{ii}).
\end{aligned}$$

Similarly

$$\begin{aligned}
\frac{\partial w_i}{\partial \log p_j} &= \frac{p_i \frac{\partial q_i}{\partial log p_j}}{m} \\
&= w_i \varepsilon_{ij}. \\
\frac{\partial w_i}{\partial \log m} &= -\frac{p_i q_i}{m} + \frac{p_i \frac{\partial q_i}{\partial \log m}}{m} \\
&= \frac{p_i q_i}{m}(\varepsilon_i - 1) \\
&= w_i(\varepsilon_i - 1).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\varepsilon_{ii} &= \frac{\partial w_i}{\partial \log p_i}\frac{1}{w_i} - 1 \\
\varepsilon_{ij} &= \frac{\partial w_i}{\partial \log p_j}\frac{1}{w_i} \\
\varepsilon_{ii} &= \frac{\partial w_i}{\partial \log m}\frac{1}{w_i} + 1.
\end{aligned}$$

Under the GCCT,

$$\begin{aligned}
\frac{\partial G_I(\underline{R}, z)}{\partial R_J} &= E[\sum_{i \in I}\sum_{j \in J} \frac{\partial g_i(\underline{r}, z)}{\partial r_j}|\underline{R}, z] \\
\frac{\partial G_I(\underline{R}, z)}{\partial z} &= E[\sum_{i \in I} \frac{\partial g_i(\underline{r}, z)}{\partial r_j}|\underline{R}, z].
\end{aligned}$$

These two identities holds because

$$E\{\sum_{i \in I}\sum_{j \in J} \frac{\partial g_i(\underline{r}, z)}{\partial r_j}|\underline{R}, z\} = E[\sum_{i \in I}\sum_{j \in J} \frac{\partial g_i(\underline{R}^* + \underline{\rho}, z)}{\partial (R_J^* + \rho_j)}|\underline{R}, z]$$

$$
\begin{aligned}
&= E[\sum_{i\in I} \frac{\partial g_i(\underline{R}^* + \underline{\rho}, z)}{\partial R_J} | \underline{R}, z] \\
&= \int \frac{\partial G_I^*(\underline{R}^* + \underline{\rho}, z)}{\partial R_J} dF(\underline{\rho}) \\
&= \frac{\partial G_I(\underline{R}, z)}{\partial R_J} \\
E\{\sum_{i\in I}\sum_{j\in J} \frac{\partial g_i(\underline{r}, z)}{\partial z} | \underline{R}, z\} &= E[\sum_{i\in I} \frac{\partial g_i(\underline{R}^* + \underline{\rho}, z)}{\partial z} | \underline{R}, z] \\
&= \int \frac{\partial G_I^*(\underline{R}^* + \underline{\rho}, z)}{\partial z} dF(\underline{\rho}) \\
&= \frac{\partial G_I(\underline{R}, z)}{\partial z}
\end{aligned}
$$

Or, equivalently, let $\hat{\varepsilon}_{IJ}$ be the elasticity estimates among the aggregated groups, let $\hat{\varepsilon}_{ij}$ be the elasticity estimates among the individual commodities, then under the GCCT,

$$
\begin{aligned}
W_I \hat{\varepsilon}_{IJ} &= E[\sum_{i\in I}\sum_{j\in J} w_i \hat{\varepsilon}_{ij} | \underline{R}, z] \\
W_I \hat{\varepsilon}_I &= E[\sum_{i\in I} w_i \hat{\varepsilon}_i | \underline{R}, z]
\end{aligned}
$$

Hence, the estimates for cross-price elasticities among groups and the income elasticities of groups are essentially the best unbiased estimates for the weighted average of the counterpart elasticities among individual commodities in these groups.

The GCCT holds under many widely used empirical models. Given the required assumption, many commonly used model specifications satisfy the generalized composite commodity theorem, such as the homothetic utility function, Deaton and Muellbauer's (1980) Almost Ideal Demand System (AIDS) and Jorgenson's Translog demand system (1982). In this paper, we will use an AIDS model for estimation.

4 MODEL SPECIFICATION

Almost Ideal Demand System (AIDS)

In the light of a model proposed by Working and Leser, Deaton and Muellbauer (1980) extended it to include the linear and quadratic effects of prices. The demand system they proposed has been widely used since their first appearance. Various revisions based on this model have been made to get better estimations. The AIDS is considerably better than other systems because

> it gives an arbitrary first order approximation to any demand system. It satisfies the axioms of choice exactly; it aggregates perfectly over consumers without invoking parallel linear Engel curves, it has a functional form which is consistent with known household-budget data; it is simple to estimate, largely avoiding non-linear estimation, and it can be used to test the restrictions of homogeneity and symmetry through linear restrictions on fixed parameters. (Deaton, Muellbauer, 1980b, p. 312)

As the first step of the model specification, a cost function is defined as

$$\log C(u,p) = a(p) + Ub(p)$$

where

$$\begin{aligned} a(p) &= \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_l \gamma^*_{kl} \log p_k \log p_l \\ b(p) &= \beta_0 \prod_k p_k^{\beta_k}. \end{aligned}$$

α's, β's, and γ^*'s are parameters. For $C(p,u)$ to be homogeneous of degree one in $\underline{p}$, these parameters must satisfy

$$\begin{aligned}\sum_k \alpha_k &= 1\\ \sum_k \gamma^*_{kl} &= 0\\ \sum_l \gamma^*_{kl} &= 0\\ \sum_k \beta_k &= 0.\end{aligned}$$

By Hotelling's Lemma,

$$\begin{aligned}\frac{\partial \log C(U,\underline{p})}{\partial \log p_i} &= \frac{\partial C(U,\underline{p})}{\partial p_i}\frac{p_i}{c}\\ &= \frac{q_i p_i}{C}\end{aligned}$$

for $\forall i = 1, 2, \cdots, n$.

From the duality concepts, we have

$$w_i = \frac{p_i q_i}{x} = \frac{\partial \log C(U,\underline{p})}{\partial \log p_i}.$$

Therefore, in AIDS model,

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_i + \beta_i U \beta_0 \prod_k p_k^{\beta_k}$$

where

$$\gamma_{ij} = \frac{1}{2}(\gamma^*_{ij} + \gamma^*_{ji}).$$

We know that

$$U\beta_0 \prod_k p_k^{\beta_k} = \log x - a(\underline{p}).$$

Thus, the AIDS model we are going to use in this paper has the following form

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_i + \beta_i \log \frac{x}{P}$$

where

$$\log P = a(\underline{p}) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_l \gamma_{kl} \log p_k \log p_l.$$

To satisfy adding-up, homogeneity and symmetry that we have discussed in Chapter 1, restrictions on the parameters should be imposed.

1. Adding-up condition implies

$$\begin{aligned}\sum_i \alpha_i &= 1 \\ \sum_i \gamma_{ij} &= 0 \ \text{ for } \ \forall j = 1, 2, \cdots, n \\ \sum_i \beta_i &= 0.\end{aligned}$$

2. Homogeneity condition implies

$$\sum_j \gamma_{ij} = 0 \ \text{ for } \ \forall i = 1, 2, \cdots, n.$$

3. Symmetry condition implies

$$\gamma_{ij} = \gamma_{ji} \ \text{ for } \ \forall i, j = 1, 2, \cdots, n.$$

AIDS model and the GCCT

In this section, we want to show that the AIDS satisfies the conditions required for the GCCT. i.e, under the AIDS model, the aggregated demand system is integrable.

If we express the AIDS model in the GCCT's notation, we have

$$z = a(\underline{r}) + b(\underline{r})U$$

where

$$\begin{aligned}a(\underline{r}) &= \alpha_0 + \sum_k \alpha_k r_k + \frac{1}{2}\sum_k \sum_l \gamma_{kl} r_k r_l \\ b(\underline{r}) &= \beta_0 exp(\sum_k r_k \beta_k)\end{aligned}$$

Thus

$$\begin{aligned}g_i(\underline{r}, z) &= \frac{\partial z}{\partial \underline{r}_i} \\ &= \frac{\partial a(\underline{r})}{\partial r_i} + \frac{\partial b(\underline{r})}{\partial r_i} \frac{[z - a(\underline{r})]}{b(\underline{r})}.\end{aligned}$$

Therefore,

$$\frac{\partial g_i(\underline{r}, z)}{\partial z} = \frac{\partial b(\underline{r})}{\partial r_i}\frac{1}{b(\underline{r})} = \beta_i.$$

Consequently, $H_{IJ} = 0$ for all group I and J, which implies H is symmetric under the AIDS model. By the GCCT, we know that the aggregated demand system under the AIDS model is integrable under the assumptions required by the GCCT.

Elasticities under AIDS model

Recall that in Chapter 3, we derived

$$\begin{aligned} \varepsilon_{ii} &= \frac{\partial w_i}{\partial \log p_i}\frac{1}{w_i} - 1 \\ \varepsilon_{ij} &= \frac{\partial w_i}{\partial \log p_j}\frac{1}{w_i} \\ \varepsilon_{ii} &= \frac{\partial w_i}{\partial \log m}\frac{1}{w_i} + 1. \end{aligned}$$

So, in the AIDS model,

$$\begin{aligned} \varepsilon_{ii} &= \frac{\gamma_{ii}}{w_i} - \beta_i + \frac{\beta_i^2}{w_i}\log\frac{m}{P} - 1 \\ \varepsilon_{ij} &= \frac{\gamma_{ij}}{w_i} - \beta_i\frac{w_j}{w_i} + \frac{\beta_i\beta_j}{w_i}\log\frac{m}{P} \\ \varepsilon_i &= \frac{\beta_i}{w_i} + 1. \end{aligned}$$

In matrix notation, denote

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \cdots & \gamma_{2n} \\ \vdots & & \ddots & & \vdots \\ \gamma_{n1} & \gamma_{n2} & \gamma_{n3} & \cdots & \gamma_{nn} \end{bmatrix}$$

$$\underline{\beta}' = (\beta_1, \beta_2, \cdots, \beta_n)' \qquad \underline{\mathbf{W}}' = (w_1, w_2, \cdots, w_n)'$$

$$[\varepsilon_{\mathbf{ij}}] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & \cdots & \varepsilon_{1n} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & \cdots & \varepsilon_{2n} \\ \vdots & & \ddots & & \vdots \\ \varepsilon_{n1} & \varepsilon_{n2} & \varepsilon_{n3} & \cdots & \varepsilon_{nn} \end{bmatrix}$$

$$\underline{\varepsilon} = (\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n)'$$

Then,

$$[\varepsilon_{\mathbf{ij}}] = \begin{bmatrix} \frac{1}{w_1} & & & \\ & \frac{1}{w_2} & & \\ & & \ddots & \\ & & & \frac{1}{w_n} \end{bmatrix} (\mathbf{\Gamma} - \underline{\beta}\underline{\mathbf{W}}' + \log \frac{m}{P} \underline{\beta \beta'}) - \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

5 DATA

So far, we have discussed the theoretical framework. It is important to go from theoretical abstraction to empirical reality because the data and estimation of the demand system allow us to test the theorem. In this chapter, we will discuss the data used in the estimation of a food demand system.

Higher level data

The data used in the estimation are annual time series data from 1970 to 1994 on at home food consumption in the United States. For estimation purpose, there are eight aggregated commodity groups: meats group (1),[1] egg group (2), dairy group (3), fats and oil group (4), fruits and vegetables group (5), cereal and bakery group (6), sweets and sugar group (7) and miscellaneous foods group (8). Detailed descriptions of each group are listed in Table 5.1.

Expenditure data come from various sources. Expenditures on different groups comes from the Consumer Expenditure Survey (CES) which is conducted by the Bureau of Labor Statistics (BLS). To be consistent, total expenditure for food at home also uses the BLS data. However, the problem is that CES only has data from 1984 to 1994.[2] Therefore, the total expenditures for food at home need to be constructed for years from 1970 to 1983. For year 1980 to 1983, relevant information can be found in *Food Spending in American Households (1980-1988)* by the Economic Research Service (ERS) of the

[1] The number in the parenthesis refers to the group number.

[2] The first CES began in 1980. BLS started collecting data annually from 1984.

Table 5.1 Description of commodity groups

commodity group	description
meats (1)	includes beef and veal, pork, other red meats, poultry and fish.
egg (2)	includes egg and related products.
dairy (3)	includes milk, cream, cheese, ice cream and related products
oil and fats (4)	includes butter, margarine, salad dressing, non-dairy cream substitutes and other fats, oils.
fruits and vegetables (5)	includes fresh fruits, fresh vegetables, juices, fresh vegetables and processed vegetables.
cereal and bakery (6)	includes cereal products, flour and prepared flour mixes, rice, pasta and cornmeal, bakery products including frozen and refrigerated.
sweets and sugar (7)	includes candy and chewing gum, sugar and artificial sweeteners, and other sweets.
miscellaneous food (8)	includes soups, snacks, nuts, seasoning, relish, sources, baby food, prepared salad and all other food that is not included in the seven other groups.

United States Department of Agriculture (USDA) (Table 10, ERS No. 824). Indeed, the data source for this publication was from the BLS. For year 1970 to 1979, we can use the concept of regression. In *Food Consumption, Prices and Expenditure 1970-1994* (ERS, No. 928), we can find total food expenditure at home. We can calculate the per capita food expenditure by dividing the total expenditure at home (Table 99, ERS No.928) by the resident number on July 1 (Table 106, ERS No. 928). Of course, these numbers don't agree with BLS's data. However, we can observe that the ratios of USDA's data to BLS's data stay almost as a constant (1.22) for years 1984-1994. Thus, for 1970 to

1979. we can simply divide USDA's data by 1.22.

Group price indices are from Table 94, ERS No. 928. The data are actually also from BLS.[3] The price indices are converted to 1984=100. Quantity data come from Table 1, ERS No. 928. Constant dollar expenditures can be obtained by multiplying the quantities by "prices" in 1984. The "prices" in 1984 are calculated by dividing the expenditures in 1984 (from Table 10, ERS No. 824, or CES of BLS[4]) by the corresponding quantities in 1984 (from Table 1, ERS No. 928). Current dollar expenditure can be obtained by multiplying the constant dollar expenditure by the prices indices (Table 94, ERS No. 928). The expenditure on miscellaneous group (8) is calculated by subtracting the expenditures for the 7 groups from the total expenditure.

The expenditures on these 8 groups are listed in Table 5.2. The corresponding budget shares are listed in Table 5.3.[5]

The price indices are used in model estimation. This is because in the demand system, we are more interested in estimating elasticities. Notice that

$$\varepsilon_{ij} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i} = \frac{\partial q_i}{\partial (p_j/p_s)}\frac{p_j/p_s}{q_i}$$

where p_s is the price in the base year. Thus, for elasticity estimation, there is no difference whether we using real prices or price indices. However, it makes life much easier by using price indices because they are directly available for groups 1 to 7. For group 8, we can construct the price index by observing that

$$\sum_{I=1}^{8}\frac{W_I}{p_I} = \frac{1}{p_{all}}$$

where W_I is the share of I th group, $I = 1, 2, \cdots, 8$. p_I is the price index for I th group, $I = 1, 2, \cdots, 8$. p_{all} is the price index for all food at home. Therefore,

$$p_8 = \frac{W_8}{\frac{1}{p_{all}} - \sum_{I=1}^{8}\frac{W_I}{p_I}}.$$

[3]Price index for miscellaneous food (8) need to be constructed. We will discuss it later.
[4]ERS used the CES data. In this paper, USDA's data were used because the former is more detail.
[5]The share for meats (1) is consolidated with the lower level data in next section.

The price indices are listed in Table 5.4.

Lower level data

To test the GCCT versus separability, a lower level system need to be estimated. The meat (1) group is chosen to illustrate this concept.

There are five commodities in meat (1) group, namely beef (11),[6] pork (12), other meats (13), poultry (14) and fish (15).

Quantity data come from Table 6, ERS No. 928. Price indices come from Table 95, ERS No. 928. A similar methodology is applied in calculating the expenditures and shares as for the group data in the previous section. The expenditures are listed in Table 5.5. Shares are shown in Table 5.6 and price indices can be found in Table 5.7.

Data for checking the assumption of the GCCT

To check the validity of the assumptions of the GCCT, we need to check

1. if the ρ_i's, i.e. the relative prices within groups and R_I's are stochastic;

2. if (1) holds, are ρ_i's and R_I's independently distributed?

Data used are time series data from 1966 to 1994 from Table 94, ERS NO. 928. Three groups are checked: meats (1) group includes beef (11), pork (12), other meats (13), poultry (14) and fish (15). Dairy (3) group includes milk (31), cheese (32) and ice cream (33). Fruits and vegetable (5) group includes fresh fruits (51), processed fruits (52), fresh vegetables (53) and processed vegetable (54). Some of the series have missing values. Group price data are list in Table 5.8 and commodity price data are listed in Table 5.9.

[6] The first number refers to the group number, the second one refers to the commodity in the group.

Table 5.2 The expenditures on commodity groups

year	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	total m
1970	103.59	9.63	40.13	8.22	36.07	33.23	8.32	72.22	297.97
1971	104.49	8.34	40.96	8.83	44.55	34.63	8.70	71.11	309.36
1972	114.84	8.10	41.70	9.18	46.88	34.29	8.95	81.10	330.66
1973	130.66	11.48	45.23	9.96	52.35	39.16	9.53	75.68	361.12
1974	134.23	11.30	51.71	13.88	60.73	50.54	14.09	76.55	404.9
1975	138.15	10.83	53.74	15.43	63.78	57.78	17.18	86.73	438.11
1976	143.54	11.57	58.18	14.14	67.32	58.09	16.00	96.34	463.71
1977	141.06	11.08	59.78	15.06	73.92	58.17	17.18	115.62	491.67
1978	160.91	10.67	64.31	16.99	78.16	62.49	18.99	122.67	536.23
1979	181.39	11.90	72.28	18.82	87.07	71.29	20.63	124.57	591.93
1980	189.76	11.46	78.63	20.37	94.95	80.18	24.84	163.55	668.47
1981	199.00	12.10	83.84	22.62	103.80	88.75	26.63	165.13	705.49
1982	202.82	11.76	87.24	22.33	108.15	94.28	26.18	180.60	735.21
1983	207.84	12.14	91.22	23.31	113.12	97.15	27.01	156.88	730.52
1984	214.83	13.56	93.86	25.03	125.14	102.20	28.68	153.30	757.11
1985	217.53	11.08	97.56	27.92	129.75	111.36	30.40	173.50	800.39
1986	226.24	11.78	97.29	27.34	132.00	118.73	30.87	131.07	776.93
1987	238.55	11.10	101.38	27.13	147.51	128.89	32.60	119.65	810.30
1988	249.63	11.04	100.55	28.44	158.86	141.10	33.75	99.36	826.46
1989	260.32	13.44	103.79	29.22	175.49	154.16	35.47	143.37	919.23
1990	275.22	13.92	114.50	31.34	187.48	170.75	38.11	118.75	955.77
1991	281.47	13.56	112.67	33.58	200.80	180.84	39.79	148.59	1019.62
1992	282.72	12.18	115.78	34.04	200.95	190.92	41.93	166.71	1057.20
1993	290.73	13.20	118.28	35.45	212.66	202.54	42.97	164.68	1094.00
1994	297.46	13.00	122.93	35.63	218.92	213.86	44.52	120.91	1084.80

Table 5.3 Expenditure shares of commodity groups

year	w1	w2	w3	w4	w5	w6	w7	w8
1970	0.34764	0.03233	0.13467	0.0276	0.12104	0.1115	0.02793	0.19729
1971	0.33776	0.02697	0.13239	0.02853	0.14401	0.11194	0.02812	0.19028
1972	0.3473	0.0245	0.12612	0.02775	0.14177	0.10369	0.02707	0.2018
1973	0.36181	0.03178	0.12525	0.02758	0.14496	0.10844	0.02639	0.17379
1974	0.33152	0.02792	0.12771	0.03428	0.14998	0.12483	0.03479	0.16898
1975	0.31534	0.02472	0.12266	0.03523	0.14558	0.13188	0.03922	0.18538
1976	0.30954	0.02495	0.12547	0.0305	0.14517	0.12528	0.03451	0.20458
1977	0.2869	0.02255	0.12159	0.03062	0.15035	0.11832	0.03494	0.23474
1978	0.30007	0.01989	0.11992	0.03169	0.14576	0.11653	0.03541	0.23072
1979	0.30644	0.0201	0.1221	0.03179	0.14709	0.12043	0.03486	0.21719
1980	0.28387	0.01715	0.11762	0.03047	0.14204	0.11995	0.03716	0.25174
1981	0.28208	0.01715	0.11884	0.03207	0.14714	0.12579	0.03775	0.23918
1982	0.27587	0.016	0.11866	0.03037	0.1471	0.12824	0.0356	0.24814
1983	0.28452	0.01662	0.12486	0.03191	0.15485	0.13299	0.03698	0.21728
1984	0.28375	0.01791	0.12397	0.03306	0.16529	0.13499	0.03788	0.20317
1985	0.27178	0.01384	0.12189	0.03489	0.16211	0.13913	0.03798	0.21839
1986	0.2912	0.01516	0.12522	0.03519	0.1699	0.15282	0.03974	0.17077
1987	0.29439	0.0137	0.12512	0.03348	0.18204	0.15907	0.04023	0.15197
1988	0.30205	0.01336	0.12166	0.03441	0.19222	0.17073	0.04083	0.12474
1989	0.2832	0.01462	0.1129	0.03179	0.19091	0.1677	0.03858	0.16029
1990	0.28796	0.01456	0.1198	0.03279	0.19616	0.17866	0.03987	0.1302
1991	0.27605	0.0133	0.1105	0.03293	0.19694	0.17737	0.03902	0.15389
1992	0.26743	0.01152	0.10951	0.0322	0.19007	0.18059	0.03967	0.16901
1993	0.26575	0.01207	0.10812	0.0324	0.19439	0.18513	0.03928	0.16287
1994	0.2742	0.01199	0.11332	0.03284	0.2018	0.19715	0.04104	0.12766

Table 5.4 Price indices for commodity groups

year	p1	p2	p3	p4	p5	p6	p7	p8
1970	42.91	60.13	44.13	36.77	35.76	35.71	29.55	33.60
1971	43.01	51.88	45.51	40.06	37.56	37.34	30.62	35.30
1972	47.18	51.51	46.20	40.43	39.36	37.54	31.10	36.12
1973	59.07	76.63	50.54	43.90	44.84	41.87	32.95	39.37
1974	60.36	76.90	59.92	62.29	52.22	54.38	50.19	46.86
1975	65.51	75.53	61.80	68.95	53.83	60.54	63.28	53.04
1976	66.11	82.49	66.83	60.32	55.25	59.19	56.10	57.39
1977	65.71	79.84	68.61	66.42	60.36	60.15	58.91	67.91
1978	76.71	75.53	73.25	72.80	67.08	65.54	66.18	72.19
1979	88.11	82.68	81.74	78.52	72.47	72.09	71.32	78.74
1980	91.38	81.21	89.73	83.77	77.67	80.75	87.69	86.84
1981	95.14	87.90	96.15	92.68	87.04	88.84	94.67	92.11
1982	99.01	85.52	97.53	90.15	91.77	92.88	94.48	95.75
1983	98.32	89.55	98.72	91.37	92.05	95.86	96.22	97.61
1984	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1985	99.60	83.41	101.88	102.16	102.55	103.85	102.52	102.39
1986	103.96	89.09	101.97	99.91	103.50	106.74	105.62	108.10
1987	110.70	83.87	104.54	101.41	112.68	110.49	107.56	107.94
1988	114.57	85.79	107.01	106.10	121.19	117.52	110.47	108.04
1989	120.32	108.62	114.12	113.70	130.56	127.43	115.70	113.86
1990	129.14	113.75	124.88	118.48	140.96	134.74	120.83	115.34
1991	132.11	111.09	123.49	123.55	147.40	140.33	125.29	119.49
1992	131.12	99.27	126.85	121.76	147.02	145.81	128.97	121.39
1993	135.38	107.33	127.74	121.95	150.43	150.72	129.26	122.85
1994	137.36	104.77	130.01	125.23	156.10	156.88	131.01	124.33

Table 5.5 Expenditures on commodities in the meat group (1)

year	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}	total (m_1)
1970	37.26	20.93	26.12	12.52	6.76	103.59
1971	38.58	20.78	25.18	12.66	7.29	104.49
1972	42.69	21.85	28.29	13.37	8.64	114.84
1973	48.18	26.13	28.40	17.84	10.10	130.66
1974	52.87	28.28	25.06	16.99	11.03	134.23
1975	55.73	28.64	23.48	18.27	12.03	138.15
1976	57.49	30.24	22.67	18.89	14.24	143.54
1977	55.54	30.02	20.77	19.23	15.51	141.06
1978	64.56	33.93	22.44	22.03	17.95	160.91
1979	73.16	39.52	24.67	24.90	19.13	181.39
1980	75.70	41.00	26.48	26.60	19.98	189.76
1981	77.13	42.94	28.44	28.55	21.93	199.00
1982	77.96	43.54	30.76	28.17	22.39	202.82
1983	78.43	45.61	30.64	28.87	24.30	207.84
1984	79.26	44.84	31.29	32.85	26.59	214.83
1985	78.40	45.43	30.37	33.67	29.67	217.53
1986	78.71	46.56	30.01	37.66	33.30	226.24
1987	78.97	50.81	30.28	40.01	38.46	238.55
1988	81.98	52.75	32.98	43.66	38.26	249.63
1989	83.14	52.67	33.58	49.86	41.08	260.32
1990	87.62	57.91	37.39	51.94	40.36	275.22
1991	88.79	60.49	38.29	53.47	40.42	281.47
1992	88.30	60.75	36.97	55.72	40.99	282.72
1993	89.55	61.89	36.68	59.68	42.93	290.73
1994	91.79	63.75	33.66	62.73	45.52	297.46

Table 5.6 Expenditure shares of commodities in the meat group (1)

year	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	total (1)
1970	0.12506	0.07023	0.08765	0.04202	0.02269	0.34764
1971	0.12472	0.06717	0.08139	0.04092	0.02357	0.33776
1972	0.12909	0.06609	0.08556	0.04043	0.02613	0.3473
1973	0.13342	0.07236	0.07865	0.0494	0.02797	0.36181
1974	0.13058	0.06985	0.0619	0.04195	0.02725	0.33152
1975	0.12721	0.06537	0.05358	0.04171	0.02746	0.31534
1976	0.12399	0.06522	0.04888	0.04074	0.03071	0.30954
1977	0.11296	0.06105	0.04224	0.03911	0.03154	0.2869
1978	0.1204	0.06327	0.04185	0.04109	0.03348	0.30007
1979	0.1236	0.06677	0.04168	0.04207	0.03231	0.30644
1980	0.11324	0.06133	0.03961	0.0398	0.02989	0.28387
1981	0.10933	0.06087	0.04031	0.04047	0.03109	0.28208
1982	0.10604	0.05922	0.04184	0.03831	0.03045	0.27587
1983	0.10736	0.06244	0.04194	0.03951	0.03326	0.28452
1984	0.10468	0.05923	0.04132	0.04339	0.03512	0.28375
1985	0.09795	0.05676	0.03794	0.04206	0.03707	0.27178
1986	0.10131	0.05993	0.03863	0.04848	0.04286	0.2912
1987	0.09746	0.06271	0.03737	0.04938	0.04747	0.29439
1988	0.09919	0.06383	0.0399	0.05282	0.0463	0.30205
1989	0.09045	0.0573	0.03653	0.05424	0.04469	0.2832
1990	0.09167	0.06059	0.03912	0.05435	0.04222	0.28796
1991	0.08708	0.05933	0.03756	0.05244	0.03964	0.27605
1992	0.08352	0.05746	0.03497	0.05271	0.03877	0.26743
1993	0.08185	0.05658	0.03353	0.05455	0.03924	0.26575
1994	0.08462	0.05877	0.03102	0.05783	0.04196	0.2742

Table 5.7 Price indices of commodities in meat group (1)

year	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}
1970	43.37	45.95	43.46	49.58	30.54
1971	45.36	41.60	43.26	49.86	33.66
1972	49.55	48.18	46.45	50.51	36.68
1973	59.42	64.07	57.84	70.83	42.05
1974	61.12	63.77	59.64	67.19	48.49
1975	61.71	78.04	63.14	74.28	52.59
1976	59.72	79.05	66.83	71.20	58.73
1977	59.32	74.80	66.43	71.67	65.07
1978	72.88	84.41	78.22	79.12	71.22
1979	92.82	85.73	89.71	83.04	78.15
1980	98.11	82.89	93.11	87.33	85.37
1981	98.90	90.59	97.10	90.87	92.49
1982	100.30	102.23	100.00	89.28	95.80
1983	98.80	101.32	99.60	90.40	96.88
1984	100.00	100.00	100.00	100.00	100.00
1985	97.91	100.30	100.70	98.97	104.88
1986	98.50	108.50	103.30	106.43	114.54
1987	105.98	117.41	109.79	104.94	126.73
1988	111.76	113.87	112.69	112.49	134.05
1989	118.94	114.57	115.88	123.67	140.10
1990	128.41	131.38	126.67	123.49	143.12
1991	132.00	135.73	131.37	122.55	144.68
1992	131.90	129.35	131.57	122.46	148.00
1993	136.69	133.30	133.67	127.59	152.78
1994	135.59	135.53	136.86	131.87	159.71

Table 5.8 Group price data used in checking assumptions

Year	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_{all}
1966	37.8	57.2	37.8	.	31.5	32.1	24.9	.	33.8
1967	36.9	47.8	39.5	34.7	31.5	32.7	25.7	.	34.1
1968	37.7	51.6	40.8	34.4	34.0	32.9	26.6	.	35.3
1969	40.8	60.7	42.2	34.5	34.4	33.9	28.0	.	37.1
1970	42.9	60.1	44.1	36.8	35.8	35.7	29.6	35.0	39.2
1971	43.0	51.9	45.5	40.1	37.6	37.3	30.6	36.4	40.4
1972	47.2	51.5	46.2	40.4	39.4	37.5	31.1	37.7	42.1
1973	59.1	76.6	50.5	43.9	44.8	41.9	32.9	41.8	48.2
1974	60.4	76.9	59.9	62.3	52.2	54.4	50.2	48.0	55.1
1975	65.5	75.5	61.8	68.9	53.8	60.5	63.3	53.7	59.8
1976	66.1	82.5	66.8	60.3	55.3	59.2	56.1	57.5	61.6
1977	65.7	79.8	68.6	66.4	60.4	60.2	58.9	67.9	65.5
1978	76.7	75.5	73.2	72.8	67.1	65.5	66.2	72.2	72.0
1979	88.1	82.7	81.7	78.5	72.5	72.1	71.3	78.5	79.9
1980	91.4	81.2	89.7	83.8	77.7	80.8	87.7	86.7	86.8
1981	95.1	87.9	96.2	92.7	87.0	88.8	94.7	92.0	93.6
1982	99.0	85.5	97.5	90.2	91.8	92.9	94.5	95.7	97.4
1983	98.3	89.6	98.7	91.4	92.1	95.9	96.2	97.6	99.4
1984	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	103.2
1985	99.6	83.4	101.9	102.2	102.6	103.8	102.5	102.4	105.6
1986	104.0	89.1	102.0	99.9	103.5	106.7	105.6	108.2	109.0
1987	110.7	83.9	104.5	101.4	112.7	110.5	107.6	107.9	113.5
1988	114.6	85.8	107.0	106.1	121.2	117.5	110.5	107.8	118.2
1989	120.3	108.6	114.1	113.7	130.6	127.4	115.7	113.7	125.1
1990	129.1	113.7	124.9	118.5	141.0	134.7	120.8	114.8	132.4
1991	132.1	111.1	123.5	123.5	147.4	140.3	125.3	118.9	136.3
1992	131.1	99.3	126.9	121.8	147.0	145.8	129.0	120.7	137.9
1993	135.4	107.3	127.7	122.0	150.4	150.7	129.3	121.9	140.9
1994	137.4	104.8	130.0	125.2	156.1	156.9	131.0	122.6	144.3

Table 5.9 Commodity price data for checking the assumptions

Year	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{32}	p_{31}	p_{33}	p_{51}	p_{52}	p_{53}	p_{54}
1966	36.1	43.2	37.8	48.8	25.0	.	.	.	30.0	.	30.3	.
1967	36.3	39.6	37.5	45.8	25.9	.	.	.	30.4	.	30.1	.
1968	37.8	39.7	38.0	47.2	26.2	.	.	.	34.4	35.6	32.2	34.0
1969	41.6	43.2	40.6	49.9	27.7	.	.	.	33.2	36.1	34.0	34.2
1970	43.4	46.0	43.5	49.6	30.5	.	.	.	33.7	36.5	36.4	35.4
1971	45.4	41.6	43.3	49.9	33.7	.	.	.	35.8	38.6	37.3	37.9
1972	49.6	48.2	46.5	50.5	36.7	.	.	.	37.7	39.7	39.6	39.6
1973	59.4	64.1	57.8	70.8	42.0	.	.	.	42.2	41.3	48.4	43.9
1974	61.1	63.8	59.6	67.2	48.5	.	.	.	45.9	47.8	51.9	62.6
1975	61.7	78.0	63.1	74.3	52.6	.	.	.	49.1	56.7	51.4	60.2
1976	59.7	79.0	66.8	71.2	58.7	.	.	.	49.0	56.4	53.6	63.3
1977	59.3	74.8	66.4	71.7	65.1	.	.	.	56.3	59.1	60.4	64.5
1978	72.9	84.4	78.2	79.1	71.2	76.2	70.9	66.6	67.2	65.5	65.2	71.1
1979	92.8	85.7	89.7	83.0	78.1	84.9	79.6	74.4	75.6	73.2	67.1	74.9
1980	98.1	82.9	93.1	87.3	85.4	92.5	87.6	84.4	80.3	78.0	73.0	80.4
1981	98.9	90.6	97.1	90.9	92.5	97.8	94.9	93.7	84.7	87.2	86.6	90.2
1982	100.3	102.2	100.0	89.3	95.8	98.5	97.2	95.6	94.0	91.9	87.1	95.1
1983	98.8	101.3	99.6	90.4	96.9	99.1	98.9	97.4	90.1	93.3	90.2	95.5
1984	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1985	97.9	100.3	100.7	99.0	104.9	101.5	101.9	103.3	110.1	104.1	95.7	101.1
1986	98.5	108.5	103.3	106.4	114.5	101.0	102.2	104.9	112.4	101.0	99.5	100.9
1987	106.0	117.4	109.8	104.9	126.7	103.2	104.5	108.5	125.0	105.1	112.4	103.7
1988	111.8	113.9	112.7	112.5	134.0	105.6	107.8	110.6	135.4	116.0	119.5	108.6
1989	118.9	114.6	115.9	123.7	140.1	113.5	116.1	116	144.3	119.7	132.3	120.2
1990	128.4	131.4	126.7	123.5	143.1	125.5	129.5	123.8	161.8	130.1	139.6	123.4
1991	132.0	135.7	131.4	122.6	144.7	121.4	131.1	125.5	183.6	125.3	142.7	124.4
1992	131.9	129.4	131.6	122.5	148.0	126.1	133.8	127.8	174.4	130.9	145.9	124.7
1993	136.7	133.3	133.7	127.6	152.8	127.7	133.6	128.6	178.8	125.8	155.6	126.6
1994	135.6	135.5	136.9	131.9	159.7	131.2	134.6	131.6	190.5	126.5	159.2	132.2

6 MODEL ESTIMATION

In this chapter, the movements of the log of the group price indices $\underline{R}_I$'s and log of relative price indices $\underline{\rho}_i$'s are examined first, to see if the assumptions of the GCCT are valid. Indeed, the assumptions are supported by empirical data. Then, we want to apply the GCCT in model estimation by using an AIDS model. Comparison of the GCCT versus separability is carried out by testing the relationship among the elasticity estimates from the model.

Testing the validity of the assumptions

Three groups are checked, meats (1), dairy (3) and fruits and vegetables (5). R_I's, $I = 1, 3, 5$, are constructed as log of the group price indices. Relative prices, ρ_{Ii}'s [1]are constructed as $\rho_{Ii} = \log p_i - \log P_I$, where $i \in I$. Figure 6.1, Figure 6.2 and Figure 6.3 show the movement of group R_I and corresponding commodity ρ_{Ii}.[2] Some of the commodities have missing observations. In such a case, the only thing we can do is to rely on the available samples.

Unit root test

From the graph, we can see that R_I's, ρ_{Ii}'s display non-stationary with stochastic trends. The unit root tests used are the augmented Dickey-Fuller test (DF test) and

[1] The first number of the subscript is the group number, the second number of the subscript refers to the commodity in the group.

[2] To get better observation of the price movement, prices are rescaled to let the first observation=100.

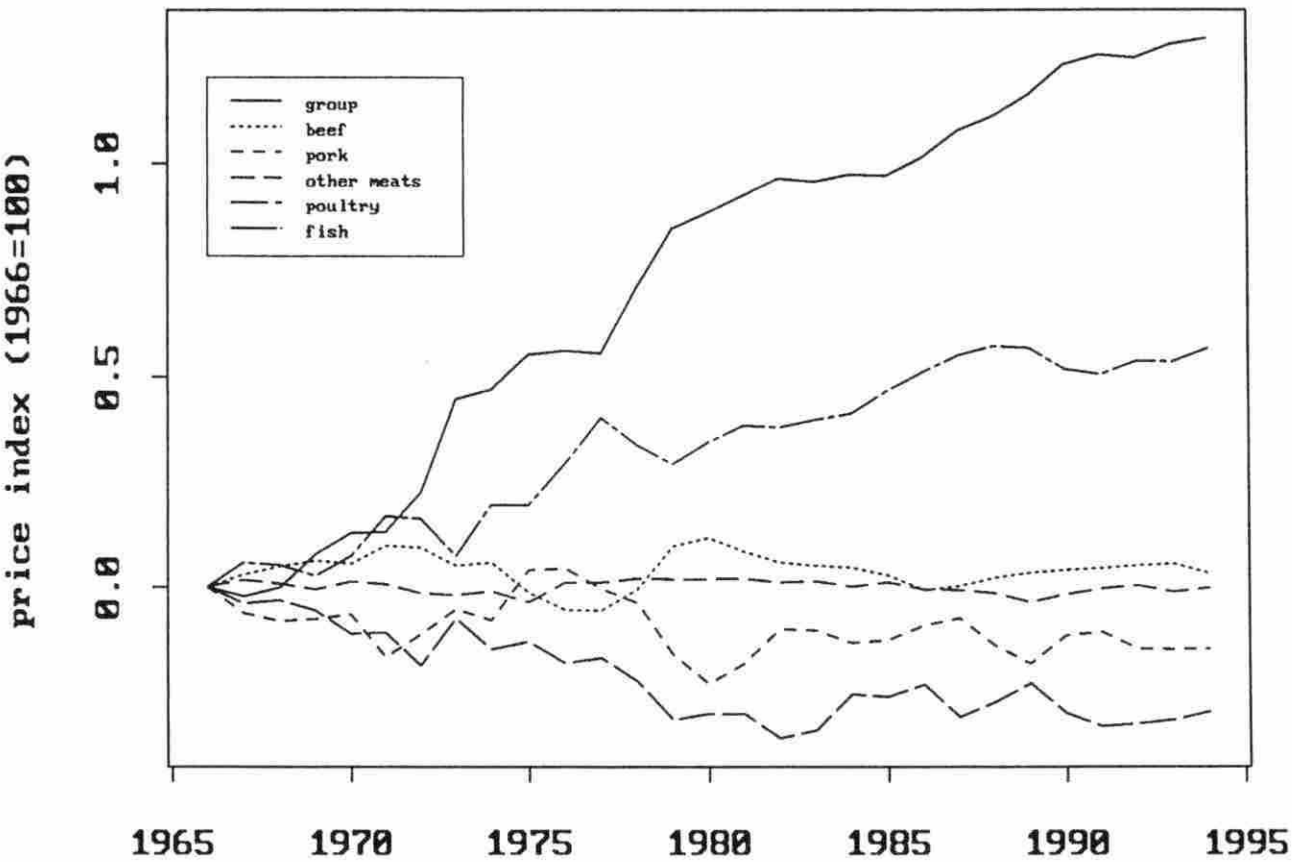

Figure 6.1 Price movement in meats group

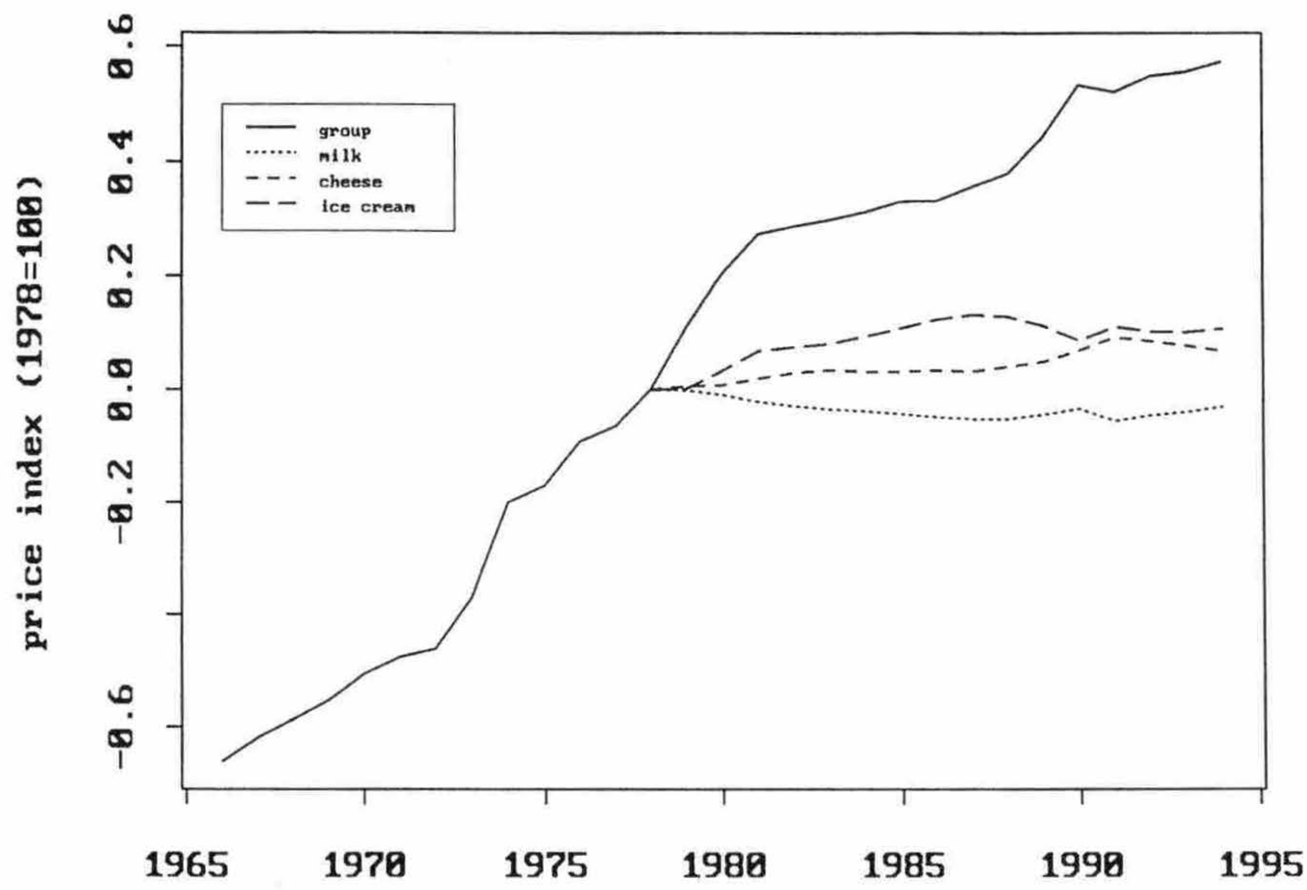

Figure 6.2 Price movement in dairy group

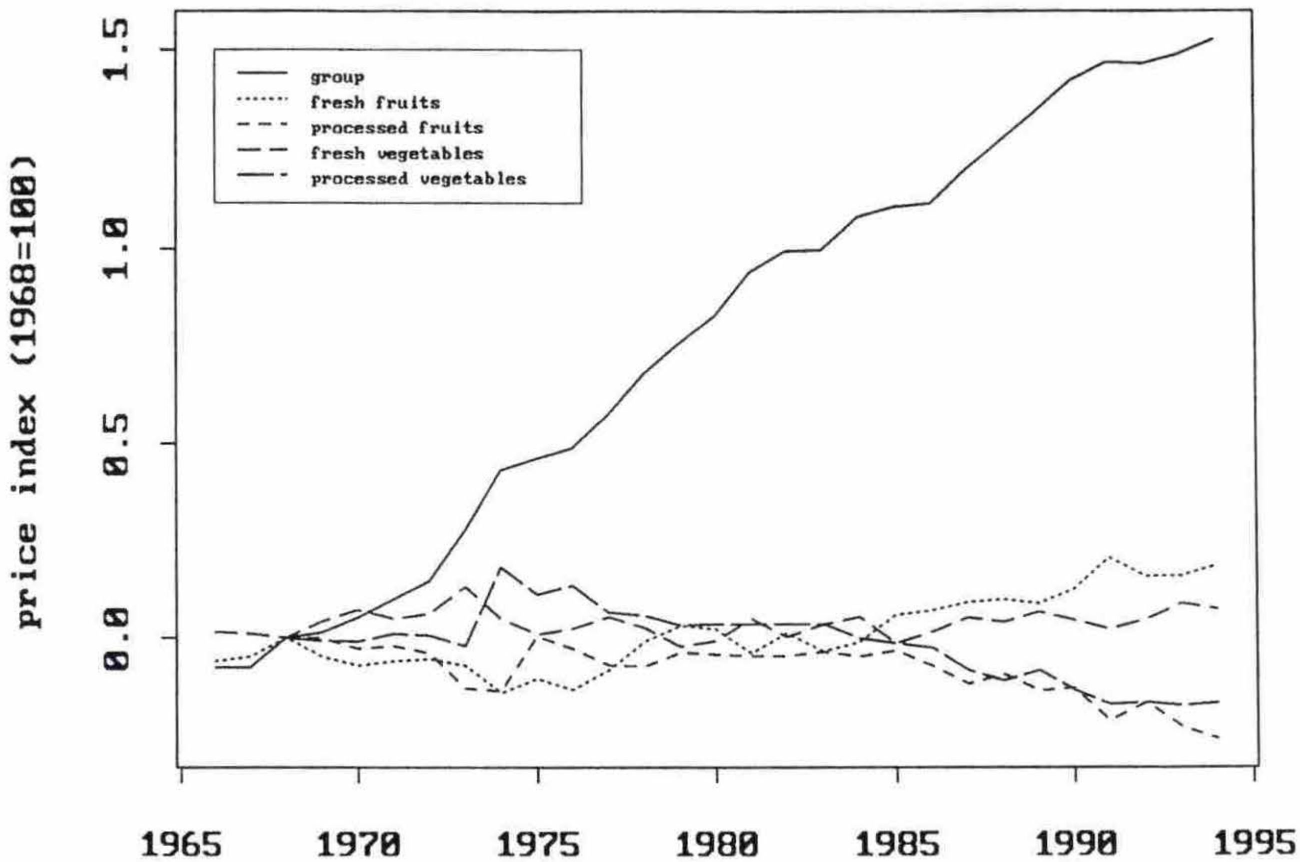

Figure 6.3 Price movement in fruits and vegetables group

Weighted Symmetry test (WS test). The augmented DF test tests the null hypothesis that the R_I's and ρ_{Ii}'s are stochastic against the alternative hypothesis of stationary. Reported P-Values for DF test is the DF t-statistics of the difference of the variable and differenced variable in regression on a constant, time trend and lags of the variable. Optimum lags can be searched automatically using the computer. Given the limited observations, we set the maximum lags to be 4. WS test has a higher power, i.e., it is more likely to reject the unit root when it is in fact false. TSP is used to perform these tests. The tested variables are R_I's, deflated R_I's and ρ_{Ii}'s. Deflated group prices are constructed by dividing the group price indices by the price index of all food at home to account for the inflation. Deflated R_I's are the log of the deflated group prices. P-Values of the two tests are listed in Table 6.1.

As we can see from Table 6.1, in both of the tests, for all of the tested variables except beef (ρ_{11}) we fail to reject the null hypothesis.

Cointegration test

Given the non-stationary, the conventional covariance or correlations can't be used to test if $\underline{\rho}$ and $\underline{R}$ are independent. Alternatively, the cointegration concept developed by Engel and Granger (1987) are applied. The Engel and Granger test (E-G test) and Johansen's test are used to test cointegration between $\underline{\rho}$ and $\underline{R}$. The null hypothesis is that the variables are not integrated.

P-values for these tests are listed in Table 6.2 to Table 6.9. E-G test1 reports the P-Values for the E-G test when ρ_{Ii}'s are the dependent variables. E-G test2 tests reports the P-Values for the E-G test when R_I's are the dependent variables. Both test the null hypothesis that $\underline{\rho}$ and $\underline{R}$ are not cointegrated. The results are listed from Table 6.2 to Table 6.9. Deflated group price indices are also tested to take the inflation into account. We can see from these tables that all of the P-Values, except the tests between the deflated R_4 and ρ_{31}, ρ_{32}, ρ_{33}, are greater than 0.05, which implies that at 95%

confidence level, we fail to reject the null hypothesis.

Johansen's test reports the P-Value for the null hypothesis that there is no cointegating vectors between the variables. Since Johansen's test includes a finite sample correction, it often has a size distortion. However, the result still shows strong evidence to support the assumption that $\underline{\rho}$ and $\underline{R}$ are not correlated even though they are both stochastic.

As shown in Table 6.2 to Table 6.9, most of the P-Values are greater than 0.05, which implies that we fail to reject the null. There are some numbers that are less than 0.05. However, none have all of the P-Values from the three tests are less than 0.05. Indeed, when there is one P-Value less than 0.05, the corresponding P-Values from the other two tests are greater than 0.05. Therefore, we could conclude that the assumption on the independence between $\underline{\rho}$ and $\underline{R}$ is valid.

Model estimation

We have shown in the previous section that the assumption of the independence between $\underline{\rho}$ and $\underline{R}$ is supported by empirical data. In this section, we want to apply the GCCT in model estimation. In chapter 4, we showed that the AIDS model satisfies the GCCT. Indeed, the AIDS model has been widely used in the estimation of food demand system in the literature. Lewbel (1991) also showed that the AIDS model fits US consumption data well. Therefore, it is used in the specification of the food demand system in this paper.

In order to compare separability to the GCCT, we need to estimate the higher level model using the aggregated (group) level data and the lower level model using the individual commodities data. Given the limited data (1970-1994 series), only commodities in the meat group are used in the lower level to save degrees of freedom.

Table 6.1 P-values of unit root test for R_I's and ρ_{Ii}'s

	P-Value for DF Test	P-Value for WS Test
Nominal R_1	0.95	0.93
Nominal R_2	0.46	0.29
Nominal R_3	0.09	0.96
Nominal R_4	0.94	0.97
Nominal R_5	0.98	0.98
Nominal R_6	0.96	0.97
Nominal R_7	0.47	0.96
Nominal R_8	0.08	0.99
Deflated R_1	0.35	0.48
Deflated R_2	0.70	0.41
Deflated R_3	0.82	0.68
Deflated R_4	0.41	0.61
Deflated R_5	0.88	0.99
Deflated R_6	0.57	0.41
Deflated R_7	0.05	0.96
Deflated R_8	0.91	0.93
ρ_{11}	0.04	0.01
ρ_{12}	0.19	0.06
ρ_{13}	0.51	0.31
ρ_{14}	0.76	0.79
ρ_{15}	0.76	0.47
ρ_{31}	0.98	0.98
ρ_{32}	0.41	0.71
ρ_{33}	0.66	0.94
ρ_{51}	0.32	0.90
ρ_{52}	0.97	0.86
ρ_{53}	0.93	0.80
ρ_{54}	0.49	0.89

Table 6.2 P-values for the cointegration test of R_1 with ρ_{Ii}'s

variable	Cointegrate Test with nominal R_1			Cointegrate Test with deflated R_1		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.07	0.95	0.05	0.21	0.76	0.04
ρ_{12}	0.27	0.95	0.06	0.26	0.58	0.05
ρ_{13}	0.45	0.88	0.17	0.76	0.76	0.06
ρ_{14}	0.22	0.49	0.92	0.87	0.63	0.95
ρ_{15}	0.36	0.9	0.1	0.72	0.66	0.31
ρ_{31}	0.96	0.1	0.26	0.69	0.72	0.13
ρ_{32}	0.76	0.13	0.66	0.92	0.95	0.04
ρ_{33}	0.93	0.11	0.09	0.47	0.52	0.05
ρ_{51}	0.16	0.74	0.93	0.54	0.6	0.96
ρ_{52}	0.8	0.57	0.39	0.99	0.48	0.22
ρ_{53}	0.41	0.9	0.88	0.89	0.63	0.93
ρ_{54}	0.37	0.64	0.0004	0.72	0.62	0.02

Table 6.3 P-values for the cointegration test of R_2 with ρ_{Ii}'s

Variables	Cointegration Test with nominal R_2			Cointegration Test with nominal R_2		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.14	0.42	0.01	0.14	0.74	0.02
ρ_{12}	0.12	0.42	0.97	0.11	0.67	0.95
ρ_{13}	0.74	0.63	0.07	0.72	0.72	0.95
ρ_{14}	0.83	0.43	0.95	0.68	0.56	0.95
ρ_{15}	0.85	0.64	0.89	0.63	0.59	0.91
ρ_{31}	0.95	0.7	0.18	0.99	0.61	0.21
ρ_{32}	0.4	0.7	0.07	0.16	0.37	0.03
ρ_{33}	0.95	0.65	0.001	0.95	0.43	0.22
ρ_{51}	0.32	0.08	0.96	0.59	0.82	0.94
ρ_{52}	0.96	0.56	0.31	0.49	0.08	0.59
ρ_{53}	0.9	0.55	0.04	0.15	0.33	0.14
ρ_{54}	0.9	0.12	0.96	0.67	0.75	0.94

Table 6.4 P-values for the cointegration test of R_3 with ρ_{Ii}'s

Variables	Cointegration Test with nominal R_3			Cointegration Test with nominal R_3		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.08	0.95	0.16	0.27	0.21	0.09
ρ_{12}	0.28	0.96	0.38	0.14	0.28	0.96
ρ_{13}	0.26	0.96	0.16	0.79	0.49	0.95
ρ_{14}	0.09	0.68	0.9	0.9	0.51	0.93
ρ_{15}	0.12	0.91	0.08	0.76	0.45	0.89
ρ_{31}	0.95	0.26	0.38	0.99	0.32	0.84
ρ_{32}	0.84	0.45	0.4	0.52	0.9	0.8
ρ_{33}	0.89	0.2	0.22	0.91	0.59	0.006
ρ_{51}	0.21	0.81	0.91	0.54	0.5	0.95
ρ_{52}	0.73	0.86	0.87	0.96	0.52	0.84
ρ_{53}	0.55	0.94	0.11	0.9	0.55	0.9
ρ_{54}	0.14	0.68	0.92	0.75	0.58	0.94

Table 6.5 P-values for the cointegration test of R_4 with ρ_{Ii}'s

Variables	Cointegration Test with nominal R_4			Cointegration Test with nominal R_4		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.08	0.96	0.05	0.06	0.83	0.02
ρ_{12}	0.35	0.96	0.13	0.39	0.86	0.07
ρ_{13}	0.52	0.97	0.64	0.7	0.75	0.47
ρ_{14}	0.34	0.77	0.92	0.79	0.62	0.96
ρ_{15}	0.24	0.87	0.72	0.67	0.67	0.19
ρ_{31}	0.93	0.61	0.63	0.96	0.01	0.09
ρ_{32}	0.69	0.98	0.39	0.24	0.05	0.03
ρ_{33}	0.94	0.56	0.48	0.95	0.01	0.86
ρ_{51}	0.29	0.91	0.15	0.56	0.55	0.96
ρ_{52}	0.81	0.81	0.02	0.96	0.58	0.24
ρ_{53}	0.55	0.92	0.06	0.81	0.48	0.04
ρ_{54}	0.38	0.61	0.24	0.91	0.56	0.02

Table 6.6 P-values for the cointegration test of R_5 with ρ_{Ii}'s

Variables	Cointegration Test with nominal R_5			Cointegration Test with nominal R_5		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.08	0.85	0.29	0.09	0.91	0.28
ρ_{12}	0.26	0.85	0.85	0.27	0.92	0.96
ρ_{13}	0.37	0.82	0.2	0.5	0.87	0.96
ρ_{14}	0.33	0.55	0.85	0.34	1	0.97
ρ_{15}	0.2	0.79	0.4	0.46	1	0.93
ρ_{31}	0.92	0.78	0.38	0.84	0.51	0.86
ρ_{32}	0.55	0.63	0.004	0.18	0.83	0.001
ρ_{33}	0.88	0.55	0.17	0.8	0.45	0.8
ρ_{51}	0.27	0.78	0.04	0.25	0.8	0.96
ρ_{52}	0.59	0.61	0.37	0.88	0.9	0.11
ρ_{53}	0.17	0.92	0.2	0.7	0.88	0.13
ρ_{54}	0.49	0.74	0.82	0.29	0.74	0.96

Table 6.7 P-values for the cointegration test of R_6 with ρ_{Ii}'s

Variables	Cointegration Test with nominal R_6			Cointegration Test with nominal R_6		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.1	0.95	0.18	0.14	0.79	0.41
ρ_{12}	0.33	0.95	0.09	0.19	0.79	0.6
ρ_{13}	0.4	0.92	0.45	0.6	0.7	0.66
ρ_{14}	0.34	0.59	0.24	0.62	0.33	0.02
ρ_{15}	0.22	0.86	0.32	0.66	0.38	0.6
ρ_{31}	0.93	0.15	0.02	0.93	0.93	0.22
ρ_{32}	0.69	0.86	0.001	0.62	0.96	0.07
ρ_{33}	0.88	0.18	0.002	0.93	0.95	0.32
ρ_{51}	0.26	0.84	0.07	0.41	0.48	0.1
ρ_{52}	0.74	0.62	0.19	0.9	0.15	0.14
ρ_{53}	0.18	0.82	0.03	0.9	0.71	0.02
ρ_{54}	0.26	0.27	0.05	0.77	0.54	0.01

Table 6.8 P-values for the cointegration test of R_7 with ρ_{Ii}'s

Variables	Cointegration Test with nominal R_7			Cointegration Test with nominal R_7		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.09	0.98	0.09	0.1	0.98	0.17
ρ_{12}	0.3	0.98	0.09	0.32	0.98	0.14
ρ_{13}	0.45	0.95	0.53	0.56	0.88	0.95
ρ_{14}	0.23	0.67	0.92	0.42	0.33	0.95
ρ_{15}	0.13	0.95	0.37	0.22	0.75	0.84
ρ_{31}	0.9	0.52	0.52	0.92	0.71	0.76
ρ_{32}	0.81	0.98	0.02	0.89	0.94	0.01
ρ_{33}	0.65	0.45	0.39	0.77	0.66	0.41
ρ_{51}	0.24	0.89	0.92	0.28	0.71	0.95
ρ_{52}	0.73	0.79	0.02	0.87	0.68	0.04
ρ_{53}	0.62	0.97	0.18	0.71	0.97	0.11
ρ_{54}	0.35	0.82	0.2	0.29	0.2	0.95

Table 6.9 P-values for the cointegration test of R_8 with ρ_{Ii}'s

Variables	Cointegration Test with nominal R_8			Cointegration Test with nominal R_8		
	E-G Test1	E-G Test2	Johansen's Test	E-G Test1	E-G Test2	Johansen's Test
ρ_{11}	0.2	0.87	0.01	0.2	0.96	0.29
ρ_{12}	0.5	0.96	0.01	0.51	0.93	0.47
ρ_{13}	0.35	0.68	0.1	0.19	0.7	0.65
ρ_{14}	0.35	0.79	0.02	0.53	0.74	0.95
ρ_{15}	0.08	0.76	0.15	0.51	0.84	0.93
ρ_{31}	0.92	0.83	0.13	0.89	0.39	0.75
ρ_{32}	0.9	0.97	0.14	0.83	0.89	0.47
ρ_{33}	0.89	0.6	0.44	0.92	0.13	0.8
ρ_{51}	0.19	0.87	0.01	0.23	0.81	0.95
ρ_{52}	0.25	0.97	0.36	0.36	0.68	0.03
ρ_{53}	0.01	0.97	0.06	0.001	0.89	0.57
ρ_{54}	0.16	0.73	0.07	0.44	0.54	0.001

Because all the shares add up to one by the construction of the data, adding-up condition is imposed by default (unrestricted model). Further restrictions on the parameters are imposed to satisfy homogeneity and symmetry (restricted model).

Four models are estimated, namely, the unrestricted higher level model, the restricted higher level model, the unrestricted lower level model and the restricted lower level model. Nonlinear Least Squares (NLS) is applied in the estimation. The software used is TSP. The linear approximation for the AIDS model is first used to obtain the starting values for the parameters.[3] Then, NLS is applied in the whole model. Because of the singularity of the system, one share equation is dropped.[4] We find that three of the four models converge, but the restricted lower level model fails to converge.[5]

The parameter estimates for higher level are listed in Table 6.10 and Table 6.13.[6] The Marshallian elasticities and Hicksian elasticities from the unrestricted higher level model are listed in Table 6.11 and Table 6.12 respectively. All the groups have negative own-price elasticities except the cereal and bakery (6) group. The income elasticities for meats (1), egg (2) and dairy (3) are negative. After imposing the homogeneity and symmetry restrictions, all of the income elasticities become positive. However, as shown in Table 6.14 and Table 6.15, some of the own-price elasticities change sign.

Parameter estimates for the lower AIDS model are listed in Table 6.16 and Table 6.19 for the unrestricted and restricted model.[7] The Marshallian elasticities are shown in Table 6.17 and Table 6.20, and the Hicksian elasticities are included in Table 6.18 and Table 6.21 for the unrestricted and restricted model respectively.

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[3]Linear approximation is to set $\log P = \sum_i w_i \log p_i$. For detail, please refer to Deaton (1980b.)

[4]Actually, it doesn't matter which one to drop since the estimates will converge to maximum likelihood estimates, which is invariance to which equation being dropped.

[5]We also tried Full Information Maximum Likelihood (FIML) method to estimate the model. It failed in both of the lower level models (unrestricted and restricted). When the estimates converge, NLS has the identical result as FIML. While NLS was applied in the restricted lower model, all the estimates for β's converge. Thus, we can use the income elasticity estimates from the lower restricted model since income elasticities only rely on β's in the AIDS model.

[6]The number in the parenthesis is the standard error of the estimate.

[7]The standard errors of the estimates are not listed because some of the estimates fail to converge.

Table 6.10 Parameter estimates from the unrestricted higher level model

groups	α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}
meats	0.049	-0.332	0.066	-0.003	-0.1222	-0.083	0.086	0.077	-0.002	-0.107
	(0.074)	(0.042)	(0.007)	(0.012)	(0.047)	(0.030)	(0.046)	(0.049)	(0.039)	(0.063)
egg	0.025	-0.026	-0.004	0.017	0.0003	-0.007	-0.007	0.010	-0.003	-0.012
	(0.011)	(0.006)	(0.004)	(0.002)	(0.006)	(0.004)	(0.006)	(0.007)	(0.005)	(0.005)
dairy	0.102	-0.170	-0.055	-0.010	0.068	-0.034	-0.002	0.055	-0.012	-0.029
	(0.047)	(0.025)	(0.012)	(0.007)	(0.028)	(0.019)	(0.028)	(0.029)	(0.024)	(0.033)
fats and oil	0.010	-0.028	-0.005	-0.004	-0.013	0.025	-0.014	0.026	-0.009	-0.004
	(0.011)	(0.007)	(0.003)	(0.002)	(0.008)	(0.005)	(0.008)	(0.009)	(0.007)	(0.006)
fruits and vegetables	0.045	-0.097	-0.034	-0.006	-0.117	0.047	0.117	0.101	-0.076	-0.018
	(0.071)	(0.048)	(0.025)	(0.012)	(0.047)	(0.035)	(0.047)	(0.057)	(0.044)	(0.033)
cereal and bakery	0.039	-0.113	-0.039	-0.002	-0.095	-0.049	0.028	0.260	-0.033	-0.061
	(0.069)	(0.043)	(0.025)	(0.011)	(0.042)	(0.032)	(0.042)	(0.053)	(0.039)	(0.032)
sweets and sugar	0.019	-0.024	-0.007	-0.001	-0.019	0.003	-0.003	0.018	0.017	-0.007
	(0.011)	(0.007)	(0.004)	(0.002)	(0.007)	(0.005)	(0.007)	(0.009)	(0.006)	(0.006)

Table 6.11 Marshallian elasticities of the unrestricted higher level model

groups	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	ε_i
meats	-0.669	-0.006	-0.405	-0.274	0.427	0.348	0.022	0.414	-0.169
egg	-0.082	-0.043	0.049	-0.358	-0.223	0.672	-0.137	0.281	-0.443
dairy	-0.326	-0.076	-0.422	-0.252	0.130	0.536	-0.065	0.696	-0.369
fats and oil	-0.087	-0.132	-0.379	-0.235	-0.317	0.847	-0.266	0.457	0.151
fruits and vegetables	-0.155	-0.033	-0.696	0.294	-0.229	0.650	-0.447	0.289	0.411
cereal and bakery	-0.221	-0.014	-0.685	-0.351	0.298	0.979	-0.227	0.115	0.166
sweets and sugar	-0.145	-0.024	-0.483	0.097	-0.016	0.522	-0.545	0.249	0.377
misc.	0.058	0.034	1.382	0.420	-1.426	-2.952	0.494	-2.460	4.883

Table 6.12 Hicksian elasticities of the unrestricted higher level model

groups	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
meats	-0.717	-0.009	-0.426	-0.280	0.399	0.326	0.016	0.379
egg	-0.208	-0.051	-0.006	-0.373	-0.296	0.612	-0.154	0.191
dairy	-0.431	-0.083	-0.467	-0.265	0.069	0.487	-0.079	0.621
fats and oil	-0.044	-0.129	-0.361	-0.230	-0.292	0.867	-0.261	0.488
fruits and vegetables	-0.038	-0.026	-0.645	0.307	-0.161	0.706	-0.431	0.373
cereal and bakery	-0.174	-0.012	-0.665	-0.345	0.325	1.001	-0.221	0.149
sweets and sugar	-0.038	-0.017	-0.437	0.109	0.047	0.573	-0.531	0.326
misc.	1.444	0.121	1.987	0.581	-0.619	-2.293	0.679	-1.468

Table 6.13 Parameter estimates from the restricted higher level model

groups	α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}
meats	2.205	-0.124	-0.001	-0.007	-0.169	-0.013	-0.244	-0.298	-0.045
	(1.013)	(0.067)	(0.154)	(0.013)	(0.092)	(0.014)	(0.098)	(0.099)	(0.016)
egg	0.131	-0.007		0.019	-0.011	-0.002	-0.020	-0.022	-0.003
	(0.098)	(0.006)		(0.002)	(0.009)	(0.002)	(0.007)	(0.010)	(0.002)
dairy	1.589	-0.094			-0.087	-0.034	-0.172	-0.162	-0.030
	(1.026)	(0.025)			(0.088)	(0.010)	(0.081)	(0.089)	(0.013)
fats and oil	0.145	-0.007				0.021	-0.012	-0.015	0.006
	(0.103)	(0.007)				(0.003)	(0.010)	(0.011)	(0.002)
fruits and vegetables	1.627	-0.941					-0.003	-0.066	-0.016
	(1.092)	(0.040)					(0.118)	(0.112)	(0.018)
cereal and bakery	1.672	-0.098						0.057	-0.022
	(1.194)	(0.039)						(0.137)	(0.021)
sweets and sugar	0.279	-0.015							0.025
	(0.174)	(0.007)							(0.004)

Table 6.14 Marshallian elasticities of the restricted higher level model

groups	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	ε_i
meats	-0.042	0.032	0.099	0.017	-0.152	-0.326	-0.039	-0.153	0.564
egg	0.507	0.124	0.012	-0.046	-0.448	-0.557	-0.060	-0.126	0.593
dairy	0.319	0.008	-0.490	-0.167	-0.147	-0.039	-0.030	0.308	0.239
fats and oil	0.084	-0.028	-0.693	-0.325	-0.008	-0.098	0.244	0.040	0.784
fruits and vegetables	-0.223	-0.046	-0.134	0.010	-0.091	0.550	0.060	-0.557	0.431
cereal and bakery	-0.602	-0.068	-0.040	-0.007	0.700	0.633	0.041	-0.928	0.271
sweets and sugar	-0.301	-0.028	-0.143	0.219	0.236	0.101	-0.237	-0.440	0.593
misc.	-0.952	-0.057	-0.175	-0.072	-0.905	-1.007	-0.180	0.182	3.167

Table 6.15 Hicksian elasticities of the restricted higher level model

groups	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
meats	0.118	0.043	0.169	0.036	-0.059	-0.250	-0.018	-0.038
egg	0.675	0.135	0.086	-0.026	-0.350	-0.477	-0.037	-0.006
dairy	0.386	0.012	-0.460	-0.159	-0.107	-0.007	-0.021	0.357
fats and oil	0.307	-0.014	-0.596	-0.299	0.121	0.008	0.273	0.200
fruits and vegetables	-0.101	-0.038	-0.080	0.024	-0.020	0.608	0.077	-0.469
cereal and bakery	-0.525	-0.063	-0.006	0.002	0.745	0.670	0.051	-0.873
sweets and sugar	-0.133	-0.018	-0.069	0.239	0.334	0.181	-0.215	-0.319
misc.	-0.054	-0.001	0.218	0.033	-0.382	-0.580	-0.060	0.825

Table 6.16 Parameter estimates from the unrestricted lower level model

commodities	α_i	β_i	γ_{i11}	γ_{i12}	γ_{i13}	γ_{i14}	γ_{i15}
beef	1.023	-0.133	-0.059	-0.029	-0.030	0.005	-0.023
pork	0.727	-0.092	-0.071	-0.047	0.031	-0.005	-0.018
o. meats	0.692	-0.072	0.040	0.013	-0.196	0.039	-0.091
poultry	-0.063	0.012	-0.003	-0.017	0.030	0.033	0.033
fish	0.412	-0.057	-0.041	-0.019	-0.051	0.029	0.025
egg	0.294	-0.037	-0.034	-0.022	0.012	0.004	-0.022
dairy	1.393	-0.195	-0.121	-0.057	-0.149	0.056	-0.102
fats and oil	0.154	-0.025	-0.021	-0.006	-0.016	0.018	-0.017
fruits and vegetables	-0.261	0.041	0.039	0.014	-0.089	0.020	0.151
cereal and bakery	-0.302	0.062	-0.038	-0.035	0.138	-0.013	0.124
sweets and sugar	0.006	0.003	-0.016	-0.009	0.023	0.005	0.015

Table 6.16 (Continued)

commodities	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}
beef	-0.034	-0.056	0.070	-0.083	-0.067	-0.038	0.209
pork	-0.016	-0.081	0.007	-0.002	-0.026	-0.007	0.209
o. meats	-0.035	0.018	-0.080	0.123	-0.084	-0.056	0.121
poultry	0.006	-0.030	-0.011	-0.004	0.040	-0.011	-0.048
fish	-0.022	-0.042	-0.004	0.014	-0.036	-0.004	0.097
egg	0.009	-0.039	-0.006	0.008	-0.013	0.002	0.058
dairy	-0.072	0.007	-0.001	0.008	-0.074	-0.038	0.361
fats and oil	-0.017	-0.006	0.032	-0.011	0.0003	-0.013	0.043
fruits and vegetables	0.006	0.019	0.089	-0.032	0.102	-0.107	-0.140
cereal and bakery	0.026	-0.071	-0.007	-0.071	0.271	-0.031	-0.229
sweets and sugar	-0.001	-0.011	0.011	-0.017	0.010	0.018	-0.022

Table 6.17 Marshallian elasticity estimates from the unrestricted lower level model

Commodities	(11)	(12)	(13)	(14)	(15)	ε_i
beef	-0.592	0.384	0.219	0.023	0.192	-0.267
pork	0.004	-0.985	1.151	-0.112	0.197	-0.553
other meats	2.308	1.209	-5.042	0.910	-1.645	-0.738
poultry	-0.301	-0.534	0.571	-0.230	0.671	1.288
fish	0.090	0.312	-0.811	0.807	0.238	-0.624
egg	-0.344	-0.166	1.506	0.195	-0.600	-1.053
dairy	0.235	0.363	-0.565	0.421	-0.314	-0.575
fats and oil	-0.071	0.208	-0.177	0.529	-0.282	0.258
fruits and vegetables	0.042	-0.048	-0.638	0.127	0.830	1.250
cereal and bakery	-0.638	-0.495	0.840	-0.086	0.772	1.459
sweets and sugar	-0.471	-0.281	0.567	0.121	0.375	1.070
misc.	-0.256	-0.208	0.488	-0.898	-1.147	3.419

Table 6.17 (Continued)

commodities	(2)	(3)	(4)	(5)	(6)	(7)	(8)
beef	-0.067	0.860	0.862	-0.843	-0.866	-0.329	-0.872
pork	0.047	0.346	0.363	-0.096	-0.711	-0.081	-1.262
other meats	-0.500	2.353	-1.655	2.912	-2.345	-1.321	-1.002
poultry	0.085	-1.018	-0.309	-0.091	0.962	-0.250	-0.449
fish	-0.308	0.583	0.142	0.319	-1.306	-0.084	-0.904
egg	-0.061	0.114	-0.010	0.365	-1.062	0.184	-1.424
dairy	-0.261	0.793	0.238	-0.006	-0.872	-0.270	-0.651
fats and oil	-0.374	0.648	0.079	-0.376	-0.119	-0.384	-0.374
fruits and vegetables	-0.012	-0.163	0.500	-1.186	0.664	-0.653	-0.281
cereal and bakery	0.096	-1.029	-0.127	-0.506	1.090	-0.238	-0.654
sweets and sugar	-0.034	-0.373	0.276	-0.433	0.289	-0.528	-0.431
misc.	0.247	-1.230	-0.866	0.435	-0.187	0.940	1.600

Table 6.18 Hicksian elasticity estimates from the unrestricted lower level model

Commodities	(11)	(12)	(13)	(14)	(15)
beef	-0.620	0.368	0.208	0.011	0.182
pork	-0.054	-1.017	1.128	-0.136	0.178
other meats	2.231	1.165	-5.073	0.878	-1.671
poultry	-0.166	-0.458	0.624	-0.174	0.716
fish	0.025	0.275	-0.836	0.780	0.216
egg	-0.454	-0.228	1.462	0.150	-0.637
dairy	0.175	0.329	-0.589	0.397	-0.334
fats and oil	-0.043	0.223	-0.166	0.540	-0.273
fruits and vegetables	0.173	0.026	-0.586	0.182	0.874
cereal and bakery	-0.485	-0.409	0.900	-0.023	0.823
sweets and sugar	-0.359	-0.218	0.611	0.168	0.413
misc.	0.102	-0.006	0.630	-0.749	-1.027

Table 6.18 (Continued)

commodities	(2)	(3)	(4)	(5)	(6)	(7)	(8)
beef	-0.072	0.827	0.853	-0.887	-0.902	-0.339	-0.926
pork	0.037	0.277	0.345	-0.187	-0.786	-0.102	-1.374
other meats	-0.513	2.262	-1.680	2.790	-2.444	-1.348	-1.151
poultry	0.108	-0.858	-0.266	0.122	1.136	-0.202	-0.188
fish	-0.319	0.506	0.122	0.216	-1.390	-0.107	-1.030
egg	-0.080	-0.016	-0.045	0.191	-1.204	0.144	-1.638
dairy	-0.271	0.722	0.219	-0.101	-0.949	-0.291	-0.768
fats and oil	-0.369	0.680	0.088	-0.333	-0.084	-0.374	-0.322
fruits and vegetables	0.010	-0.008	0.541	-0.979	0.833	-0.606	-0.026
cereal and bakery	0.122	-0.848	-0.078	-0.265	1.286	-0.183	-0.358
sweets and sugar	-0.015	-0.240	0.311	-0.256	0.433	-0.488	-0.213
misc.	0.308	-0.806	-0.753	1.000	0.275	1.069	2.294

Table 6.19 Parameter estimates from the restricted lower level model

commodities	α_i	β_i	γ_{i11}	γ_{i12}	γ_{i13}	γ_{i14}	γ_{i15}
beef	1.438	-0.080	-0.087	-0.060	0.127	-0.028	-0.028
pork	1.002	-0.057		-0.045	0.065	-0.029	-0.020
other meats	-0.209	0.015			-0.123	0.010	-0.023
poultry	0.260	-0.013				0.031	-0.006
fish	0.629	-0.036					0.029
egg	0.258	-0.014					
dairy	2.338	-0.133					
fats and oil	0.159	-0.007					
fruits and vegetables	0.986	-0.050					
cereal and bakery	0.718	-0.035					
sweets and sugar	0.244	-0.012					

Table 6.19 (Continued)

commodities	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}
beef	-0.009	-0.160	0.018	-0.165	-0.066	-0.034
pork	-0.010	-0.110	0.006	-0.070	-0.037	-0.018
other meats	0.001	0.097	-0.034	0.056	-0.115	0.016
poultry	-0.001	-0.058	-0.013	0.024	0.021	0.002
fish	-0.011	-0.095	0.000	-0.029	-0.033	-0.010
egg	0.013	-0.028	0.001	-0.021	-0.017	-0.005
dairy		-0.223	-0.042	-0.128	-0.078	-0.031
fats and oil			0.025	-0.015	-0.004	0.004
fruits and vegetables				0.123	0.042	-0.002
cereal and bakery					0.209	-0.002
sweets and sugar						0.027

Table 6.20 Marshallian elasticity estimates from the restricted lower level model

Commodities	(11)	(12)	(13)	(14)	(15)	ε_i
beef	-2.131	-0.792	1.320	-0.298	-0.406	0.234
pork	-1.380	-2.038	1.228	-0.529	-0.518	0.045
other meats	3.226	1.681	-4.027	0.264	-0.480	1.370
poultry	-0.767	-0.761	0.279	-0.293	-0.203	0.701
fish	-1.185	-0.870	-0.507	-0.219	-0.353	-0.014
egg	-0.825	-0.787	0.162	-0.103	-0.742	0.198
dairy	-1.703	-1.197	0.924	-0.507	-0.961	-0.072
fats and oil	0.465	0.107	-1.004	-0.399	-0.039	0.776
fruits and vegetables	-1.113	-0.509	0.378	0.137	-0.232	0.699
cereal and bakery	-0.592	-0.345	-0.820	0.149	-0.294	0.741
sweets and sugar	-1.018	-0.572	0.456	0.049	-0.329	0.672
misc.	3.223	2.214	-1.127	0.305	1.493	3.076

Table 6.20 (Continued)

commodities	(2)	(3)	(4)	(5)	(6)	(7)	(8)
beef	-0.142	-2.056	0.165	-1.681	-0.695	-0.352	6.833
pork	-0.235	-2.520	0.084	-1.312	-0.693	-0.342	8.211
other meats	0.049	2.595	-0.824	1.399	-2.765	-3.850	-5.196
poultry	-0.052	-1.544	-0.302	0.522	0.468	0.042	1.911
fish	-0.374	-3.400	-0.010	-0.976	-1.027	-0.329	9.265
egg	-0.342	-2.113	0.063	-1.268	-1.015	-0.323	7.097
dairy	-0.300	-3.540	-0.354	-1.180	-0.714	-0.295	9.899
fats and oil	0.024	-1.431	-0.233	-0.479	-0.148	0.100	2.261
fruits and vegetables	-0.146	-0.980	-0.093	-0.298	0.230	-0.026	1.955
cereal and bakery	-0.144	-0.757	-0.035	0.275	0.531	-0.023	1.315
sweets and sugar	-0.161	-1.056	0.091	-0.109	-0.071	-0.290	2.338
misc.	0.574	5.650	0.292	1.197	0.558	0.345	-6.804

Table 6.21 Hicksian elasticity estimates from the restricted lower level model

Commodities	(11)	(12)	(13)	(14)	(15)
beef	-2.107	-0.778	1.330	-0.287	-0.398
pork	-1.375	-2.035	1.230	-0.527	-0.517
other meats	3.369	1.763	-3.970	0.324	-0.432
poultry	-0.693	-0.720	0.308	-0.263	-0.178
fish	-1.186	-0.871	-0.508	-0.220	-0.354
egg	-0.804	-0.775	0.170	-0.095	-0.735
dairy	-1.711	-1.201	0.921	-0.510	-0.964
fats and oil	0.546	0.153	-0.972	-0.366	-0.012
fruits and vegetables	-1.040	-0.467	0.406	0.167	-0.208
cereal and bakery	-0.514	-0.301	-0.790	0.181	-0.268
sweets and sugar	-0.948	-0.532	0.484	0.078	-0.305
misc.	3.545	2.396	-1.000	0.438	1.601

Table 6.21 (Continued)

commodities	(2)	(3)	(4)	(5)	(6)	(7)	(8)
beef	-0.138	-2.027	0.173	-1.642	-0.663	-0.343	6.881
pork	-0.234	-2.515	0.085	-1.304	-0.687	-0.341	8.220
other meats	0.074	2.765	-0.779	1.626	-2.580	-3.798	-4.917
poultry	-0.039	-1.458	-0.279	0.638	0.563	0.068	2.053
fish	-0.375	-3.402	-0.011	-0.978	-1.028	-0.329	9.262
egg	-0.339	-2.088	0.069	-1.235	-0.989	-0.316	7.137
dairy	-0.301	-3.549	-0.356	-1.191	-0.724	-0.297	9.884
fats and oil	0.037	-1.334	-0.207	-0.350	-0.043	0.130	2.419
fruits and vegetables	-0.134	-0.894	-0.070	-0.182	0.324	0.000	2.097
cereal and bakery	-0.131	-0.665	-0.011	0.397	0.631	0.005	1.465
sweets and sugar	-0.149	-0.973	0.113	0.002	0.020	-0.264	2.475
misc.	0.629	6.032	0.394	1.706	0.974	0.462	-6.179

Under unrestricted lower level model, all the Marshallian own-price elasticities are negative except fish (15), dairy (3), fats and oil (4), cereal and bakery (6) and miscellaneous (8). Beef (11), pork (12), other meats (13), fish (15), egg (2) and dairy (3) have negative income elasticities. Given the result from the higher level model, it comes without surprise. We can also observe that beef (11) is gross substitute for pork (12), other meats (13), poultry (14) and fish (15), which agrees with our intuition and expectation.

After imposing the homogeneity and symmetry conditions on the lower level model, all of the income elasticities are positive except fish (15) and dairy (3). Most of the own-price elasticities are negative except fish (15), cereal and bakery (6), sweets and sugar (7) and miscellaneous (8). All the goods are necessities except other meats (13) and miscellaneous (8).

The abnormal signs of some of the elasticities are also cited in the literature. Indeed, Huang (1993) estimated a complete system of US demand for food (ERS, No. 1821). From his work, we can also find a positive own-price elasticities for rice, negative income elasticities in other meats, turkey and fish.

There are several reasons which would account for these unconventional signs. First, the system estimated here is only a food demand system. We treat the expenditure on food as if it were income. Second, the aggregated data used could cause problem because of their poor quality. Third, the expenditure data from 1970 to 1980 were constructed because we could not find the ready-to-use data. Given the relatively short time series, this will affect the estimation. Fourth, we only used the AIDS model to estimate. There could exist a better model which fits the data better.

Testing separability versus the GCCT

Under separability, there is no aggregation bias when summing demands for individual goods to obtain group demands. So group demand errors will just equal to the sum of the errors of individual goods in the group.[8] Since

$$W_I = \sum_{i \in I} w_i$$

and

$$W_I = G_I(\underline{R}, z) + e_I$$

$$w_i = g_i(\underline{r}, z) + e_i.$$

Given

$$\sum_{i \in I} e_i = e_I,$$

we can conclude that

$$G_I(\underline{R}, z) = \sum_{i \in I} g_i(\underline{r}, z).$$

Thus, under separability,

$$\frac{\partial G_I(\underline{R}, z)}{\partial R_J} = \sum_{i \in I} \sum_{j \in J} \frac{\partial g_i(\underline{r}, z)}{\partial R_j}$$

$$\frac{\partial G_I(\underline{R}, z)}{\partial z} = \sum_{i \in I} \sum_{j \in J} \frac{\partial g_i(\underline{r}, z)}{\partial z}$$

In the previous section, what we estimated is essentially $G_I(\underline{R}, z)$ from the higher level model and $g_i(\underline{r}, z)$ from the lower level model. In this particular food demand model, only the commodities in the meats group(1) are included in the lower level demand system. Therefore, for this model, we can test separability versus the GCCT simply by examining the relationship between elasticities for the individual goods in the meats group(1) from the lower level model and the elasticities in the higher level model. That is, under separability,

$$\varepsilon_{1J} W1 = \sum_{i \in group1} \varepsilon_{iJ} w_i \text{ for } J = 2, 3, \cdots, 8$$

$$\varepsilon_1 W_1 = \sum_{i \in group1} \varepsilon_i w_i.$$

[8]The uppercase subscription refers to the higher level model and the lowercase subscription refers to the lower level model.

The left hand of equations are elasticities and share in the higher level model, and the right hand of the equations are elasticities and share in the lower level model. Given the estimates from the lower level model, we can treat the RHS as if they were constants as estimated from the lower level model and test in the higher level model to see if the LHS equal to these constants. Because of the abnormal signs of the elasticities, we will just check the cross-price elasticities between meats (1) and fruits and vegetables (4), and the income elasticities in the meats (1) group. As discussed above, the restricted lower level model failed to converge. However, the estimates for β's converged. So, we can still go ahead and check the relationships among the income elasticities in the meats(1) group between the restricted lower level model and restricted higher level model. Therefore, for the unrestricted models, the hypothesis tested is

$$\begin{aligned} H_0 : \varepsilon_{15} W1 &= \sum_{i \in meats(1)} \varepsilon_{i5} w_i = 0.03369 \\ \varepsilon_1 W1 &= \sum_{i \in meats(1)} \varepsilon_i w_i = -0.6979 \end{aligned}$$

A Wald Test is used for this joint test. The test statistic has an asymptotic χ^2 distribution with 2 degrees of freedom. The value is 248.62 , P-Value is close to zero. Thus, we reject H_0 (separability) in favor of H_1 (the GCCT). For the restricted models, the hypothesis tested is

$$H_0 : \varepsilon_1 W1 = \sum_{i \in meats(1)} \varepsilon_i w_i = 0.1137$$

The test statistic has an asymptotic χ^2 distribution with 1 degrees of freedom. The value is 0.4838, P-Value is 0.4867. Thus, in this case, we fail to reject H_0.

This is a very interesting result. The unrestricted models tended to reject separability in favor of the GCCT, while the restricted model failed to reject separability. However, because the estimation problem presented in the restricted lower level model, we were not able to include the hypothesis tests among cross-price elasticities. This could lead to the high P-Value of the test statistic.

7 CONCLUSION

The Generalized Composite Commodity Theorem provides a new rationalization for aggregation over commodities. This paper provides an empirical study for the GCCT by an application in a US food demand system.

The movement of the price movements from US food demand system strongly support the assumptions proposed by the GCCT. That is, the relative price of a commodity within the group is independently distributed with the group price.

Comparison between the GCCT and separability is carried out by testing the relationship between the elasticities of the group level model and that of the commodity level model. Indeed, the statistical test is in favor of the GCCT over separability in unrestricted models. As long as the restricted model is concerned, we are not able to draw a conclusion because of the estimation problem.

However, the model used in this paper is an AIDS model. In order to find a model which fits the data better, we may also try different model specifications which satisfy the GCCT.

In this paper, we only used the meat group (1) in the estimation of the lower level model. Testing the separability versus the GCCT was also done base on this group only. We may also try to include more commodities if data were available.

Another issue addressed in this paper is the limited sample problem. The time series we used are from 1970 to 1994. Given the limited size of data set, we should be conservative with the Wald Test which applies asymptotic properties of large samples. One possible solution is use the bootstrapping method proposed by Efron.

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